Hindawi
Advances in High Energy Physics
Volume 2020, Article ID 6438730, 7 pages
https://doi.org/10.1155/2020/6438730

Research Article

Light Tetraquark State Candidates

Zhi-Gang Wang

Department of Physics, North China Electric Power University, Baoding 071003, China

Correspondence should be addressed to Zhi-Gang Wang; zgwang@aliyun.com

Received 3 March 2020; Accepted 2 April 2020; Published 9 May 2020

Guest Editor: Jajati K. Nayak

In this article, we study the axialvector-diquark-axialvector-antidiquark type scalar, axialvector, tensor, and vector $ss\bar{s}\bar{s}$ tetraquark states with the QCD sum rules. The predicted mass $m_X = 2.08 \pm 0.12$ GeV for the axialvector tetraquark state is in excellent agreement with the experimental value $(2062.8 \pm 13.1 \pm 4.2)$ MeV from the BESIII collaboration and supports assigning the new X state to be a $ss\bar{s}\bar{s}$ tetraquark state with $J^{PC} = 1^{+-}$. The predicted mass $m_X = 3.08 \pm 0.11$ GeV disfavors assigning $\phi(2170)$ or $Y(2175)$ to be the vector partner of the new X state. As a byproduct, we obtain the masses of the corresponding $qq\bar{q}\bar{q}$ tetraquark states. The light tetraquark states lie in the region about 2 GeV rather than 1 GeV.

1. Introduction

Recently, the BESIII collaboration studied the process $J/\psi \longrightarrow \phi\eta\eta'$ and observed a structure X in the $\phi\eta'$ mass spectrum [1]. The fitted mass and width are $m_X = (2002.1 \pm 27.5 \pm 15.0)$ MeV and $\Gamma_X = (129 \pm 17 \pm 7)$ MeV, respectively, with assumption of the spin-parity $J^P = 1^-$, the corresponding significance is 5.3σ, while the fitted mass and width are $m_X = (2062.8 \pm 13.1 \pm 4.2)$ MeV and $\Gamma_X = (177 \pm 36 \pm 20)$ MeV, respectively, with assumption of the spin-parity $J^P = 1^+$, the corresponding significance is 4.9σ. The X state was observed in the $\phi\eta'$ decay model rather than in the $\phi\eta$ decay model; they may contain a large $ss\bar{s}\bar{s}$ component; in other words, it may have a large tetraquark component. In Ref. [2], Wang et al. assign the X state to be the second radial excitation of $h_1(1380)$. In Ref. [3], Cui et al. assign X to be the partner of the tetraquark state $Y(2175)$ with the $J^{PC} = 1^{+-}$.

We usually assign the lowest scalar nonet mesons $\{f_0(500), a_0(980), \kappa_0(800), f_0(980)\}$ to be tetraquark states and assign the higher scalar nonet mesons $\{f_0(1370), a_0(1450), K_0^*(1430), f_0(1500)\}$ to be the conventional $^3P_0$ quark-antiquark states [4–6]. In Ref. [7], we take the nonet scalar mesons below 1 GeV as the two-quark-tetraquark mixed states and study their masses and pole residues with the QCD sum rules in detail and observe that the dominant Fock components of the nonet scalar mesons below 1 GeV are conventional two-quark states. The light tetraquark states may lie in the region about 2 GeV rather than lie in the region about 1 GeV.

In this article, we take the axialvector diquark operators as the basic constituents to construct the tetraquark current operators to study the scalar (S), axialvector (A), tensor (T), and vector (V) tetraquark states with the QCD sum rules and explore the possible assignments of the new X state. We take the axialvector diquark operators as the basic constituents because the favored configurations from the QCD sum rules are the scalar and axialvector diquark states [8–10]; the current operators or quark structures chosen in the present work differ from that in Ref. [3] completely.

The article is arranged as follows: we derive the QCD sum rules for the masses and pole residues of the $ss\bar{s}\bar{s}$ tetraquark states in Section 2; in Section 3, we present the numerical results and discussions; Section 4 is reserved for our conclusion.

2. QCD Sum Rules for the $ss\bar{s}\bar{s}$ Tetraquark States

We write down the two-point correlation functions $\Pi_{\mu\nu\alpha\beta}(p)$ and $\Pi(p)$ firstly

$$\Pi_{\mu\nu\alpha\beta}(p) = i\int d^4x e^{ip\cdot x}\left\langle 0 \mid T\left\{J_{\mu\nu}(x)J^{\dagger}_{\alpha\beta}(0)\right\} \mid 0\right\rangle,$$
$$\Pi(p) = i\int d^4x e^{ip\cdot x}\left\langle 0 \mid T\left\{J_{0}(x)J^{\dagger}_{0}(0)\right\} \mid 0\right\rangle, \quad (1)$$

where $J_{\mu\nu}(x) = J_{2,\mu\nu}(x)$, $J_{1,\mu\nu}(x)$,

$$J_{2,\mu\nu}(x) = \frac{\varepsilon^{ijk}\varepsilon^{imn}}{\sqrt{2}}\left\{s^{Tj}(x)C\gamma_\mu s^k(x)\bar{s}^m(x)\gamma_\nu C\bar{s}^{Tn}(x) + s^{Tj}(x)C\gamma_\nu s^k(x)\bar{s}^m(x)\gamma_\mu C\bar{s}^{Tn}(x)\right\},$$
$$J_{1,\mu\nu}(x) = \frac{\varepsilon^{ijk}\varepsilon^{imn}}{\sqrt{2}}\left\{s^{Tj}(x)C\gamma_\mu s^k(x)\bar{s}^m(x)\gamma_\nu C\bar{s}^{Tn}(x) - s^{Tj}(x)C\gamma_\nu s^k(x)\bar{s}^m(x)\gamma_\mu C\bar{s}^{Tn}(x)\right\},$$
$$J_0(x) = \varepsilon^{ijk}\varepsilon^{imn}s^{Tj}(x)C\gamma_\mu s^k(x)\bar{s}^m(x)\gamma^\mu C\bar{s}^{Tn}(x), \quad (2)$$

where i, j, k, m, and n are color indexes and C is the charge conjugation matrix. Under charge conjugation transform $\widehat{C}$, the currents $J_{\mu\nu}(x)$ and $J_0(x)$ have the properties

$$\widehat{C}\,J_{2,\mu\nu}(x)\,\widehat{C}^{-1} = +J_{2,\mu\nu}(x),$$
$$\widehat{C}\,J_{1,\mu\nu}(x)\,\widehat{C}^{-1} = -J_{1,\mu\nu}(x), \quad (3)$$
$$\widehat{C}\,J_{0}(x)\,\widehat{C}^{-1} = +J_{0}(x).$$

The doubly strange diquark operators

$$s^{Tj}C\Gamma s^k = \frac{1}{2}\left(s^{Tj}C\Gamma s^k - s^{Tk}C\Gamma s^j\right) = \frac{1}{2}\varepsilon^{ijk}s^{Tj}C\Gamma s^k \quad (4)$$

with $\Gamma = \gamma_\mu$, $\sigma_{\mu\nu}$ in color antitriplet $\bar{3}_c$ and

$$s^{Tj}C\Gamma s^k = \frac{1}{2}\left(s^{Tj}C\Gamma s^k + s^{Tk}C\Gamma s^j\right) \quad (5)$$

with $\Gamma = 1$, γ_5, $\gamma_\mu\gamma_5$ in color sextet 6_c satisfy Fermi-Dirac statistics. On the other hand, the scattering amplitude for one-gluon exchange is proportional to

$$\left(\frac{\lambda^a}{2}\right)_{ij}\left(\frac{\lambda^a}{2}\right)_{kl} = -\frac{1}{3}\left(\delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj}\right) + \frac{1}{6}\left(\delta_{ij}\delta_{kl} + \delta_{il}\delta_{kj}\right), \quad (6)$$

where

$$\varepsilon_{mik}\varepsilon_{mjl} = \delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj}. \quad (7)$$

λ^a is the Gell-Mann matrix. The negative sign in front of the antisymmetric antitriplet $\bar{3}_c$ indicates that the interaction is attractive, which favors formation of the diquarks in color antitriplet. The positive sign in front of the symmetric sextet 6_c indicates that the interaction is repulsive, which disfavors formation of the diquarks in color sextet. The diquark states which couple potentially to the $s^{Tj}Cs^k$, $s^{Tj}C\gamma_5 s^k$, and $s^{Tj}C\gamma_\mu\gamma_5 s^k$ operators in color sextet 6_c are expected to have larger masses than the diquark states which couple potentially to the $s^{Tj}C\gamma_\mu s^k$ and $s^{Tj}C\sigma_{\mu\nu}s^k$ operators in color antitriplet $\bar{3}_c$. We prefer the diquark operators in color antitriplet $\bar{3}_c$ to the diquark operators in color sextet 6_c in constructing the tetraquark current operators. Up to now, the scalar and axialvector diquark states in color antitriplet $\bar{3}_c$ have been studied with the QCD sum rules [8–10]. In our previous studies, we observed that the pseudoscalar and vector diquark states in color antitriplet $\bar{3}_c$ are not favored configurations and cannot lead to stable QCD sum rules, which are not included in Ref. [8]. The tensor diquark states, which have both $J^P = 1^+$ and 1^- components, have not been studied with the QCD sum rules yet. We can draw the conclusion tentatively that the most favored quark configuration is the axialvector diquark operator $\varepsilon^{ijk}s^{Tj}C\gamma_\mu s^k$. In Ref. [3], Cui et al. choose the pseudoscalar diquark operator in color sextet 6_c and vector antidiquark operator in color antisextet $\bar{6}_c$ and axialvector diquark operator in color antitriplet $\bar{3}_c$ and tensor antidiquark operator in color triplet 3_c to construct the axialvector currents to study the axialvector tetraquark states. In Ref. [11], we choose the color octet-octet type vector four-quark current to study $Y(2175)$; Fierz rearrangement of this current cannot lead to a diquark-antidiquark type tensor component. In the present work, we choose the axialvector diquark (antidiquark) operators in color antitriplet $\bar{3}_c$ (triplet 3_c) to construct the tensor current, which is expected to couple potentially to the lowest tetraquark states, to study both the axialvector and vector tetraquark states. The quark configuration in the present work differs completely from that in Ref. [3] and Ref. [11]; it is interesting to study the new quark configuration. Furthermore, the conclusion of the present work differs completely from that of Ref. [3].

At the hadronic side, we can insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators $J_{\mu\nu}(x)$ and $J_0(x)$ into the correlation functions $\Pi_{\mu\nu\alpha\beta}(p)$ and $\Pi(p)$ to obtain the hadronic representation [11–14]. After isolating the ground state contributions of the scalar, axialvector, vector, and tensor tetraquark states, we get the results

$$\Pi_{2,\mu\nu\alpha\beta}(p) = \frac{\lambda^2_{X_T}}{m^2_{X_T} - p^2}\left(\frac{\tilde{g}_{\mu\alpha}\tilde{g}_{\nu\beta} + \tilde{g}_{\mu\beta}\tilde{g}_{\nu\alpha}}{2} - \frac{\tilde{g}_{\mu\nu}\tilde{g}_{\alpha\beta}}{3}\right) + \cdots = \Pi_{2^+}(p)\left(\frac{\tilde{g}_{\mu\alpha}\tilde{g}_{\nu\beta} + \tilde{g}_{\mu\beta}\tilde{g}_{\nu\alpha}}{2} - \frac{\tilde{g}_{\mu\nu}\tilde{g}_{\alpha\beta}}{3}\right) + \cdots,$$

$$\begin{aligned}\Pi_{1,\mu\nu\alpha\beta}(p) &= \frac{\tilde{\lambda}_{X_A}^2}{m_{X_A}^2-p^2}\left(p^2 g_{\mu\alpha}g_{\nu\beta}-p^2 g_{\mu\beta}g_{\nu\alpha}-g_{\mu\alpha}p_\nu p_\beta\right.\\ &\quad\left.-g_{\nu\beta}p_\mu p_\alpha+g_{\mu\beta}p_\nu p_\alpha+g_{\nu\alpha}p_\mu p_\beta\right)\\ &\quad+\frac{\tilde{\lambda}_{X_V}^2}{m_{X_V}^2-p^2}\left(-g_{\mu\alpha}p_\nu p_\beta-g_{\nu\beta}p_\mu p_\alpha+g_{\mu\beta}p_\nu p_\alpha\right.\\ &\quad\left.+g_{\nu\alpha}p_\mu p_\beta\right)+\cdots=\Pi_{1^+}\left(p^2\right)\left(p^2 g_{\mu\alpha}g_{\nu\beta}-p^2 g_{\mu\beta}g_{\nu\alpha}\right.\\ &\quad\left.-g_{\mu\alpha}p_\nu p_\beta-g_{\nu\beta}p_\mu p_\alpha+g_{\mu\beta}p_\nu p_\alpha+g_{\nu\alpha}p_\mu p_\beta\right)\\ &\quad+\Pi_{1^-}\left(p^2\right)\left(-g_{\mu\alpha}p_\nu p_\beta-g_{\nu\beta}p_\mu p_\alpha+g_{\mu\beta}p_\nu p_\alpha\right.\\ &\quad\left.+g_{\nu\alpha}p_\mu p_\beta\right),\\ \Pi(p) &= \Pi_{0^+}\left(p^2\right)=\frac{\lambda_{X_S}^2}{m_{X_S}^2-p^2}+\cdots,\end{aligned} \tag{8}$$

where $\tilde{g}_{\mu\nu}=g_{\mu\nu}-(p_\mu p_\nu/p^2)$; the subscripts 2^+, 1^+, 1^-, and 0^+ denote the spin-parity J^P of the corresponding tetraquark states. The pole residues λ_X and $\tilde{\lambda}_X$ are defined by

$$\begin{aligned}\langle 0\mid J_{2,\mu\nu}(0)\mid X_T(p)\rangle &= \lambda_{X_T}\,\varepsilon_{\mu\nu},\\ \langle 0\mid J_{1,\mu\nu}(0)\mid X_A(p)\rangle &= \tilde{\lambda}_{X_A}\,\varepsilon_{\mu\nu\alpha\beta}\,\varepsilon^\alpha p^\beta,\\ \langle 0\mid J_{1,\mu\nu}(0)\mid X_V(p)\rangle &= \tilde{\lambda}_{X_V}\left(\varepsilon_\mu p_\nu-\varepsilon_\nu p_\mu\right),\\ \langle 0\mid J_0(0)\mid X_S(p)\rangle &= \lambda_{X_S},\end{aligned} \tag{9}$$

where $\varepsilon_{\mu\nu}$ and ε_μ are the polarization vectors of the tetraquark states.

Now we contract s quarks in the correlation functions with the Wick theorem; there are four s-quark propagators; if two s-quark lines emit a gluon by itself and the other two s-quark lines contribute a quark pair by itself, we obtain operator $GG\bar{s}s\bar{s}s$, which is of order $\mathcal{O}(\alpha_s^k)$ with $k=1$ and of dimension 10. In this article, we take into account that the vacuum condensates up to dimension 10 and $k\le 1$ in a consistent way. For the technical details, one can consult Refs. [7, 15]. Once the analytical expressions of the QCD spectral densities are obtained, we take the quark-hadron duality below the continuum thresholds s_0 and perform Borel transform with respect to the variable $P^2=-p^2$ to obtain the QCD sum rules:

$$\lambda_X^2\,\exp\left(-\frac{m_X^2}{T^2}\right)=\int_0^{s_0}ds\,\rho(s)\,\exp\left(-\frac{s}{T^2}\right), \tag{10}$$

where $\rho(s)=\rho_S(s)$, $\rho_A(s)$, $\rho_V(s)$, and $\rho_T(s)$.

$$\begin{aligned}\rho_S(s) &= \frac{s^4}{3840\pi^6}-\frac{13s\,m_s\langle\bar{s}g_s\sigma Gs\rangle}{384\pi^4}+\frac{2s\langle\bar{s}s\rangle^2}{3\pi^2}-\frac{17\langle\bar{s}s\rangle\langle\bar{s}g_s\sigma Gs\rangle}{48\pi^2}\\ &\quad+\frac{s^2}{192\pi^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle+\frac{19m_s\langle\bar{s}s\rangle}{96\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle-\frac{16m_s\langle\bar{s}s\rangle^3}{3}\delta(s)\\ &\quad+\frac{\langle\bar{s}g_s\sigma Gs\rangle^2}{192\pi^2}\delta(s)-\frac{\langle\bar{s}s\rangle^2}{24}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\delta(s),\end{aligned} \tag{11}$$

$$\begin{aligned}\rho_A(s) &= \frac{s^4}{11520\pi^6}-\frac{s^2\,m_s\langle\bar{s}s\rangle}{12\pi^4}+\frac{s\,m_s\langle\bar{s}g_s\sigma Gs\rangle}{9\pi^4}+\frac{4s\,\langle\bar{s}s\rangle^2}{9\pi^2}\\ &\quad-\frac{5\langle\bar{s}s\rangle\langle\bar{s}g_s\sigma Gs\rangle}{18\pi^2}-\frac{s^2}{2304\pi^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\\ &\quad+\frac{3m_s\langle\bar{s}s\rangle}{64\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle-\frac{32m_s\langle\bar{s}s\rangle^3}{9}\delta(s)\\ &\quad-\frac{2\langle\bar{s}s\rangle^2}{27}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\delta(s),\end{aligned} \tag{12}$$

$$\begin{aligned}\rho_V(s) &= \frac{s^4}{11520\pi^6}+\frac{s^2\,m_s\langle\bar{s}s\rangle}{12\pi^4}-\frac{7s\,m_s\langle\bar{s}g_s\sigma Gs\rangle}{72\pi^4}-\frac{2s\langle\bar{s}s\rangle^2}{9\pi^2}\\ &\quad+\frac{5\langle\bar{s}s\rangle\langle\bar{s}g_s\sigma Gs\rangle}{18\pi^2}+\frac{s^2}{768\pi^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\\ &\quad-\frac{79m_s\langle\bar{s}s\rangle}{1728\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle+\frac{16m_s\langle\bar{s}s\rangle^3}{9}\delta(s)\\ &\quad-\frac{2\langle\bar{s}s\rangle^2}{81}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\delta(s)-\frac{\langle\bar{s}g_s\sigma Gs\rangle^2}{18\pi^2}\delta(s),\end{aligned} \tag{13}$$

$$\begin{aligned}\rho_T(s) &= \frac{s^4}{5376\pi^6}-\frac{3s^2\,m_s\langle\bar{s}s\rangle}{20\pi^4}+\frac{29s\,m_s\langle\bar{s}g_s\sigma Gs\rangle}{96\pi^4}+\frac{8s\langle\bar{s}s\rangle^2}{9\pi^2}\\ &\quad-\frac{37\langle\bar{s}s\rangle\langle\bar{s}g_s\sigma Gs\rangle}{48\pi^2}-\frac{11s^2}{1920\pi^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\\ &\quad+\frac{43m_s\langle\bar{s}s\rangle}{864\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle-\frac{64m_s\langle\bar{s}s\rangle^3}{9}\delta(s)\\ &\quad-\frac{4\langle\bar{s}s\rangle^2}{27}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\delta(s),\end{aligned} \tag{14}$$

and $\lambda_{X_{A/V}}=m_{X_{A/V}}\tilde{\lambda}_{X_{A/V}}$.

We derive equation (10) with respect to $\tau=1/T^2$, then obtain the QCD sum rules for the masses of the tetraquark states through a fraction

$$m_X^2=-\frac{\int_0^{s_0}ds(d/d\tau)\rho(s)\,\exp\,(-\tau s)}{\int_0^{s_0}ds\rho(s)\,\exp\,(-\tau s)}. \tag{15}$$

3. Numerical Results and Discussions

We take the standard values of the vacuum condensates $\langle\bar{q}q\rangle=-(0.24\pm0.01\,\text{GeV})^3$, $\langle\bar{q}g_s\sigma Gq\rangle=m_0^2\langle\bar{q}q\rangle$, $m_0^2=(0.8\pm0.1)\,\text{GeV}^2$, $\langle\bar{s}s\rangle=(0.8\pm0.1)\langle\bar{q}q\rangle$, $\langle\bar{s}g_s\sigma Gs\rangle=m_0^2\langle\bar{s}s\rangle$, and

Table 1: The Borel parameters, continuum threshold parameters, pole contributions, contributions of the vacuum condensates of dimension 10, masses, and pole residues of the tetraquark states, where the subscripts S, A, T, and V denote the scalar, axialvector, tensor, and vector tetraquark states, respectively.

	T^2 (GeV^2)	$\sqrt{s_0}$ (GeV)	Pole	$\lvert D(10)\rvert$	m_X(GeV)	λ_X (10^{-2}GeV^5)
$ss\bar{s}\bar{s}_S$	$1.4-1.8$	2.65 ± 0.10	$(40-73)\%$	$\ll1\%$	2.08 ± 0.13	2.73 ± 0.56
$ss\bar{s}\bar{s}_A$	$1.5-1.9$	2.65 ± 0.10	$(41-72)\%$	$<1\%$	2.08 ± 0.12	1.87 ± 0.34
$ss\bar{s}\bar{s}_T$	$1.5-1.9$	2.75 ± 0.10	$(41-72)\%$	$<1\%$	2.22 ± 0.11	3.02 ± 0.53
$ss\bar{s}\bar{s}_V$	$2.1-2.7$	3.60 ± 0.10	$(42-73)\%$	$\leq1\%$	3.08 ± 0.11	6.47 ± 1.07
$qq\bar{q}\bar{q}_S$	$1.2-1.6$	2.40 ± 0.10	$(40-76)\%$	$\ll1\%$	1.86 ± 0.11	1.95 ± 0.38
$qq\bar{q}\bar{q}_A$	$1.3-1.7$	2.40 ± 0.10	$(40-73)\%$	$\leq1\%$	1.87 ± 0.10	1.30 ± 0.22
$qq\bar{q}\bar{q}_T$	$1.4-1.8$	2.65 ± 0.10	$(42-74)\%$	$\leq1\%$	2.13 ± 0.10	2.58 ± 0.42
$qq\bar{q}\bar{q}_V$	$1.9-2.5$	3.40 ± 0.10	$(41-74)\%$	$\leq2\%$	2.86 ± 0.11	4.94 ± 0.93

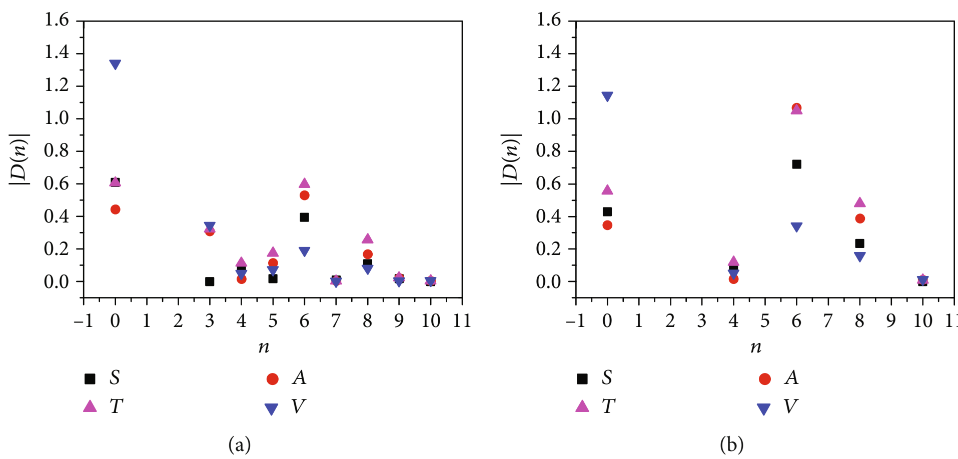

Figure 1: The absolute contributions of the vacuum condensates of dimension n for the central values of the input parameters in the operator product expansion, where the S, A, T, and V denote the scalar, axialvector, tensor, and vector tetraquark states, respectively, (a) and (b) denote the $ss\bar{s}\bar{s}$ and $qq\bar{q}\bar{q}$ quark constituents, respectively.

$\langle\alpha_s GG/\pi\rangle = (0.012\pm0.004)\,\text{GeV}^4$ at the energy scale $\mu=1$ GeV [12–14, 16] and choose the $\overline{MS}$ mass $m_s(\mu=2\,\text{GeV}) = 0.095\pm0.005\,\text{GeV}$ from the Particle Data Group [17] and evolve the s-quark mass to the energy scale $\mu=1\,\text{GeV}$ with the renormalization group equation; furthermore, we neglect the small u and d quark masses.

We choose suitable Borel parameters and continuum threshold parameters to warrant the pole contributions (PC) are larger than 40%, i.e.,

$$\text{PC} = \frac{\int_0^{s_0} ds\,\rho(s)\exp\left(-s/T^2\right)}{\int_0^{\infty} ds\,\rho(s)\exp\left(-s/T^2\right)} \geq 40\%, \tag{16}$$

and convergence of the operator product expansion. The contributions of the vacuum condensates $D(n)$ in the operator product expansion are defined by

$$D(n) = \frac{\int_0^{s_0} ds\,\rho_n(s)\exp\left(-s/T^2\right)}{\int_0^{s_0} ds\,\rho(s)\exp\left(-s/T^2\right)}, \tag{17}$$

where the subscript n in the QCD spectral density $\rho_n(s)$ denotes the dimension of the vacuum condensates. We choose the values $|D(10)|\sim1\%$ to warrant the convergence of the operator product expansion. In Table 1, we present the ideal Borel parameters, continuum threshold parameters, pole contributions, and contributions of the vacuum condensates of dimension 10. In Figure 1, we plot the absolute contributions of the vacuum condensates of dimension n for the central values of the input parameters in the operator product expansion. Although in some cases the contributions of the perturbative terms $D(0)$ are not the dominant contributions, the contributions of the vacuum condensates of dimensions 6 and 8 are very large; the hierarchy $|D(6)|\gg|D(8)|$ warrants the good convergent behavior of the operator product expansion; furthermore, the contributions $D(7)$, $D(9)$, and $D(10)$ are very small. From Table 1 and Figure 1, we can see that the pole dominance is well satisfied and the operator product expansion is well converged; we expect to make reliable predictions.

We take into account all uncertainties of the input parameters and obtain the values of the masses and pole

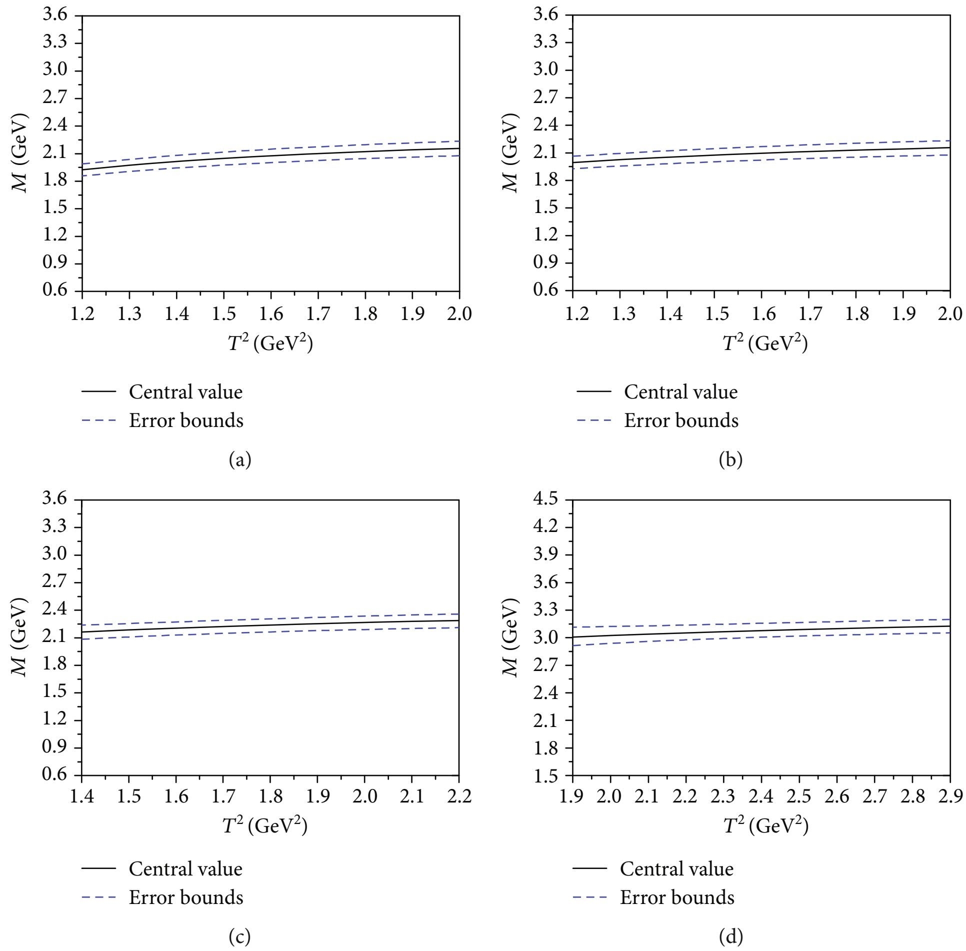

FIGURE 2: The masses with variations of the Borel parameters T^2, where (a), (b), (c), and (d) denote the scalar, axialvector, tensor, and vector tetraquark states, respectively.

residues of the $ss\bar{s}\bar{s}$ tetraquark states, which are shown explicitly in Figure 2 and Table 1. In this article, we have assumed that the energy gaps between the ground state and the first radial state are about 0.6 GeV [18–20]. In Figure 2, we plot the masses of the scalar, axialvector, tensor, and vector $ss\bar{s}\bar{s}$ tetraquark states with variations of the Borel parameters at larger regions than the Borel windows shown in Table 1. From the figure, we can see that there appear platforms in the Borel windows.

From Table 1, we can see that the uncertainties of the masses δM_X are small, while the uncertainties of the pole residues $\delta\lambda_X$ are large, for example, $\delta M_X/M_X = 6\%$ and $\delta\lambda_X/\lambda_X = 21\%$ for the scalar $ss\bar{s}\bar{s}$ tetraquark state. We obtain the tetraquark masses from a fraction, see equation (14); the uncertainties originating from the input parameters in the numerator and denominator are almost canceled out with each other, so the net uncertainties of the tetraquark masses are very small. In this article, we have neglected the perturbative $\mathcal{O}(\alpha_s)$ corrections. For the traditional two-quark light mesons, the perturbative $\mathcal{O}(\alpha_s)$ corrections amount to multiplying the perturbative terms with a factor $1 + (11/3)(\alpha_s/\pi)$ for the $J^{PC} = 0^{+-}$, 0^{++} mesons, $1 + (\alpha_s/\pi)$ for the $J^{PC} = 1^{--}$, 1^{++}, 1^{+-} mesons, and $1 - (\alpha_s/\pi)$ for the $J^{PC} = 2^{++}$ mesons [14]. Now we estimate the possible uncertainties due to neglecting the perturbative $\mathcal{O}(\alpha_s)$ corrections by multiplying the perturbative terms with a factor $1 + (-1 \sim 4)(\alpha_s/\pi)$. The additional uncertainties δM_X and $\delta\lambda_X$ are shown in Table 2. From the table, we can see again that the uncertainties of the mass δM_X are small, while the uncertainties of the pole residues $\delta\lambda_X$ are large, for example, $\delta M_X/M_X = {}^{+2\%}_{-1\%}$ and $\delta\lambda_X/\lambda_X = {}^{+23\%}_{-7\%}$ for the scalar $ss\bar{s}\bar{s}$ tetraquark state. In the QCD sum rules for the X, Y, Z states, which are excellent candidates for the compact tetraquark states or loosely bound molecular states, the uncertainties of the masses are less than or about 6% [21]. Ref. [21] is the most recent review.

The predicted mass $m_X = 2.08 \pm 0.12$ GeV for the axialvector tetraquark state is in excellent agreement with the experimental value $(2062.8 \pm 13.1 \pm 4.2)$ MeV from the BESIII collaboration [1], which supports assigning the new X state to be an axialvector-diquark-axialvector-antidiquark type $ss\bar{s}\bar{s}$ tetraquark state. The predicted mass $m_X = 3.08 \pm 0.11$ GeV for the vector tetraquark state lies above the experimental value of the mass of $\phi(2170)$ or $Y(2175)$, $m_\phi = 2188 \pm 10$ MeV, from the Particle Data Group, and disfavors assigning $\phi(2170)$ or $Y(2175)$ to be the vector partner of the new X state. If $\phi(2170)$ have a tetraquark component, it may

TABLE 2: The possible uncertainties induced by the perturbative $\mathcal{O}(\alpha_s)$ corrections, where the subscripts S, A, T, and V denote the scalar, axialvector, tensor, and vector tetraquark states, respectively.

	δm_X (GeV)	$\delta\lambda_X$ (10^{-2}GeV^5)
$ss\bar{s}\bar{s}_S$	$^{+0.04}_{-0.02}$	$^{+0.64}_{-0.18}$
$ss\bar{s}\bar{s}_A$	$^{+0.03}_{-0.02}$	$^{+0.33}_{-0.09}$
$ss\bar{s}\bar{s}_T$	$^{+0.03}_{-0.01}$	$^{+0.63}_{-0.18}$
$ss\bar{s}\bar{s}_V$	$^{+0.03}_{-0.06}$	$^{+1.62}_{-0.45}$
$qq\bar{q}\bar{q}_S$	$^{+0.04}_{-0.01}$	$^{+0.35}_{-0.10}$
$qq\bar{q}\bar{q}_A$	$^{+0.03}_{-0.01}$	$^{+0.18}_{-0.05}$
$qq\bar{q}\bar{q}_T$	$^{+0.03}_{-0.01}$	$^{+0.51}_{-0.14}$
$qq\bar{q}\bar{q}_V$	$^{+0.02}_{-0.02}$	$^{+1.27}_{-0.37}$

have color octet-octet component [11]. As a byproduct, we obtain the masses and pole residues of the corresponding $qq\bar{q}\bar{q}$ tetraquark states, which are shown in Table 1. The present predictions can be confronted to the experimental data in the future.

Now we perform Fierz rearrangement to the currents both in the color and Dirac-spinor spaces

$$\begin{aligned}
J_0 &= 2\bar{s}s\,\bar{s}s + 2\bar{s}i\gamma_5 s\,\bar{s}i\gamma_5 s + \bar{s}\gamma_\alpha s\,\bar{s}\gamma^\alpha s - \bar{s}\gamma_\alpha\gamma_5 s\,\bar{s}\gamma^\alpha\gamma_5 s, \\
J_{1,\mu\nu} &= \sqrt{2}\left\{ i\bar{s}s\,\bar{s}\sigma_{\mu\nu}s - \bar{s}\sigma_{\mu\nu}\gamma_5 s\,\bar{s}i\gamma_5 s + i\varepsilon_{\mu\nu\alpha\beta}\bar{s}\gamma^\alpha\gamma_5 s\,\bar{s}\gamma^\beta s \right\}, \\
J_{2,\mu\nu} &= \frac{1}{\sqrt{2}}\left\{ 2\bar{s}\gamma_\mu\gamma_5 s\,\bar{s}\gamma_\nu\gamma_5 s - 2\bar{s}\gamma_\mu s\,\bar{s}\gamma_\nu s + 2g^{\alpha\beta}\bar{s}\sigma_{\mu\alpha}s\,\bar{s}\sigma_{\nu\beta}s \right. \\
&\quad + g_{\mu\nu}\left(\bar{s}s\,\bar{s}s + \bar{s}i\gamma_5 s\,\bar{s}i\gamma_5 s + \bar{s}\gamma_\alpha s\,\bar{s}\gamma^\alpha s - \bar{s}\gamma_\alpha\gamma_5 s\,\bar{s}\gamma^\alpha\gamma_5 s \right. \\
&\quad \left.\left. - \frac{1}{2}\bar{s}\sigma_{\alpha\beta}s\,\bar{s}\sigma^{\alpha\beta}s\right)\right\}.
\end{aligned} \tag{18}$$

The diquark-antidiquark type currents can be rearranged into currents as special superpositions of color singlet-singlet type currents, which couple potentially to the meson-meson pairs or molecular states; the diquark-antidiquark type tetraquark states can be taken as special superpositions of meson-meson pairs and embody the net effects. The decays to their components are Okubo-Zweig-Iizuka supper-allowed; we can search for those tetraquark states in the decays

$$\begin{aligned}
X_S &\longrightarrow \eta'\eta',\ f_0(980)f_0(980),\ \phi(1020)\phi(1020), \\
X_{A/V} &\longrightarrow f_0(980)h_1(1380),\ \phi(1020)\eta',\ \phi(1020)\phi(1020), \\
X_T &\longrightarrow \eta'\eta',\ f_0(980)f_0(980),\ \phi(1020)\phi(1020).
\end{aligned} \tag{19}$$

4. Conclusion

In this article, we construct the axialvector-diquark-axialvector-antidiquark type currents to interpolate the scalar, axialvector, tensor, and vector $ss\bar{s}\bar{s}$ tetraquark states, then calculate the contributions of the vacuum condensates up to dimension 10 in the operator product expansion and obtain the QCD sum rules for the masses and pole residues of those tetraquark states. The predicted mass $m_X = 2.08 \pm 0.12$ GeV for the axialvector tetraquark state is in excellent agreement with the experimental value, $m_X = (2062.8 \pm 13.1 \pm 4.2)$ MeV, from the BESIII collaboration and supports assigning the new X state to be an axialvector-diquark-axialvector-antidiquark type $ss\bar{s}\bar{s}$ tetraquark state. The predicted mass $m_X = 3.08 \pm 0.11$ GeV for the vector tetraquark state lies above the experimental value of the mass of $\phi(2170)$, $m_\phi = 2188 \pm 10$ MeV, from the Particle Data Group and disfavors assigning $\phi(2170)$ to be the vector partner of the new X state. As a byproduct, we also obtain the masses and pole residues of the corresponding $qq\bar{q}\bar{q}$ tetraquark states. The present predictions can be confronted to the experimental data in the future.

Data Availability

All data included in this manuscript are available upon request by contacting with the corresponding author.

Conflicts of Interest

The author declares that they have no conflicts of interest.

Acknowledgments

This work is supported by the National Natural Science Foundation (Grant number 11775079).

References

[1] BESIII Collaboration, "Observation and study of the decay $J/\psi \longrightarrow \phi\eta\eta'$," *Physics Review D*, vol. 99, no. 11, Article ID 112008, 2019.
[2] L. M. Wang, J. Z. Wang, S. Q. Luo, J. He, and X. Liu, "Studying X(2100) hadronic decays and predicting its pion and kaon induced productions," *Physics Review D*, vol. 101, no. 3, Article ID 034021, 2020.
[3] E. L. Cui, H. M. Yang, H. X. Chen, W. Chen, and C. P. Shen, "QCD sum rule studies of $ss\bar{s}\bar{s}$ tetraquark states with $J^{PC} = 1^{+-}$," *The European Physical Journal C*, vol. 79, no. 3, Article ID 232, 2019.
[4] F. E. Close and N. A. Törnqvist, "Scalar mesons above and below 1-GeV," *Journal of Physics G*, vol. 28, no. 10, pp. R249–R267, 2002.
[5] C. Amsler and N. A. Tornqvist, "Mesons beyond the naive quark model," *Physics Reports*, vol. 389, no. 2, pp. 61–117, 2004.
[6] L. Maiani, F. Piccinini, A. D. Polosa, and V. Riquer, "New Look at Scalar Mesons," *Physical Review Letters*, vol. 93, no. 21, Article ID 212002, 2004.
[7] Z. G. Wang, "Analysis of the scalar nonet mesons with QCD sum rules," *The European Physical Journal C*, vol. 76, no. 8, Article ID 427, 2016.
[8] Z. G. Wang, "Analysis of the light-flavor scalar and axial-vector diquark states with QCD sum rules," *Communications in Theoretical Physics*, vol. 59, no. 4, pp. 451–456, 2013.

[9] H. G. Dosch, M. Jamin, and B. Stech, “Diquarks, QCD sum rules, and weak decays,” *Zeitschrift für Physik C Particles and Fields*, vol. 42, no. 1, pp. 167–173, 1989.

[10] M. Jamin and M. Neubert, “Diquark decay constants from QCD sum rules,” *Physics Letters B*, vol. 238, no. 2-4, pp. 387–394, 1990.

[11] Z. G. Wang, “Analysis of Y(2175) as a tetraquark state with QCD sum rules,” *Nuclear Physics A*, vol. 791, no. 1-2, pp. 106–116, 2007.

[12] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, “QCD and resonance physics. theoretical foundations,” *Nuclear Physics B*, vol. 147, no. 5, pp. 385–447, 1979.

[13] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, “QCD and resonance physics. applications,” *Nuclear Physics B*, vol. 147, no. 5, pp. 448–518, 1979.

[14] L. J. Reinders, H. Rubinstein, and S. Yazaki, “Hadron Properties from QCD Sum Rules,” *Physics Reports*, vol. 127, no. 1, pp. 1–97, 1985.

[15] Z. G. Wang and T. Huang, “Analysis of theX(3872),Zc(3900), andZc(3885)as axial-vector tetraquark states with QCD sum rules,” *Physical Review D*, vol. 89, no. 5, Article ID 054019, 2014.

[16] P. Colangelo and A. Khodjamirian, “QCD sum rules, a modern perspective,” *At The Frontier of Particle Physics: Handbook of QCD*, vol. 3, pp. 1495–1576, 2001.

[17] C. Patrignani, D. H. Weinberg, C. L. Woody et al., “Review of particle physics,” *Chinese Physics*, vol. 40, no. 10, Article ID 100001, 2016.

[18] Z. G. Wang, “Analysis of the Z (4430) as the First Radial Excitation of the Zc (3900),” *Communications in Theoretical Physics*, vol. 63, no. 3, pp. 325–330, 2015.

[19] Z. G. Wang, “Scalar tetraquark state candidates: X (3915), X (4500) and X (4700),” *The European Physical Journal C*, vol. 77, no. 2, Article ID 78, 2017.

[20] Z. G. Wang, “Reanalysis of the X (3915), X (4500) and X (4700) with QCD sum rules,” *The European Physical Journal A*, vol. 53, no. 2, Article ID 19, 2017.

[21] R. M. Albuquerque, J. M. Dias, K. P. Khemchandani et al., “QCD sum rules approach to the X, Y and Z states,” *Journal of Physics G*, vol. 46, no. 9, Article ID 093002, 2019.