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Early Mathematics Learning and Teaching in Chinese Preschools: A Content Analysis of Teaching Reference Books for Preschool Teachers

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Abstract: Since the 1980s, Chinese children have consistently been found to perform well in mathematics in cross-national studies of academic achievement. This study is one of the first to systematically examine Chinese children's early mathematics learning experiences through a content analysis of eight sets of preschool teacher's teaching reference books that are commonly used in China. A total of 2038 mathematics activities were selected from these classic teaching reference books and analyzed in terms of (i) suggested teaching approaches, (ii) the mathematical concepts covered, and (iii) their integration of a problem-solving component. The results showed that early mathematics teaching in Chinese preschools emphasize the mastery of key mathematical concepts and skills, while less attention is paid to the application of these key concepts in real-life and non-routine problem-solving settings. Collective teaching was found to be the major teaching mode for early mathematics activities in Chinese preschools, with teachers playing a leading role in the collective teaching activities described in the reference books. However, the teacher's role is not described in detail in the learning corner activities, and few activities were found to integrate a problem-solving component. The implications of these findings for early mathematics education and the need for a more comprehensive review of mathematics education from the pre-primary to secondary stages are discussed.

Keywords: early mathematics education; teaching reference books; content analysis; China

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1. Introduction

Mathematics is one of the most important components of children's education and is of great importance in early learning and development [1]. In China, mathematics is highly valued by both society at large and the educational system [2]. Chinese students have consistently been found to perform at an excellent standard based on mathematics measures in cross-national studies of school achievement, such as the Trends in International Mathematics and Science Study [3,4] and the Programme for International Student Assessment (PISA) [5–7].

When examining the findings of these international comparative studies more closely, it becomes clear that Chinese students' outstanding mathematics performance primarily occurs in the subject-related mathematics test, which focuses on the mastery of key mathematical concepts and skills, such as number, shape, and measurement, and their applications. In contrast, their performance at problem solving, which places less emphasis on subject knowledge and more on identifying problem-solving scenarios, selecting relevant mathematics knowledge from knowledge reserves, and applying such knowledge to solve the problem identified, is less outstanding than their performance in the subject-related mathematics test. In fact, in this area, Chinese students are below the OECD average [8,9]. This observation suggests that there may be gaps in the teaching of problem solving in the Chinese curriculum. To determine whether such gaps exist and where they may lie, we started at the beginning of Chinese children's formal education and examined their early mathematics learning experiences through a systematic content analysis of the teaching reference books that are commonly used at the preschool level. The goal of the analysis was to identify ways to better support Chinese students' mathematics learning performance.

A content analysis of a text-based curriculum describes and analyzes topics that are covered in the classroom and is a popular method used in mathematics education research [10,11]. Chinese preschool teachers traditionally use teaching reference books in the planning and implementation of their daily teaching activities [12]. Therefore, the contents in these reference books would be reflective of, and provide insights into, what is taught in early mathematics and which mathematics activities are implemented in Chinese preschools, especially given preschool teachers' faithful use of these reference books in their daily teaching [13]. In this study, we conducted a content analysis of eight sets of commonly used teaching reference books for preschool teachers.

1.1. Content and Pedagogy in Mathematics Education in Chinese Preschools

In recent decades, mathematics education in Chinese preschools has undergone significant changes. Before the 1980s, mathematics was known as "computing" and was treated as an individual subject [14,15]. Preschool teachers usually designed multiple mathematical activities to help children master the "two basics", which referred to the basic mathematical concepts and skills. The principle of the "two basics" defined the characteristics of Chinese mathematics for a long time. The "basic concepts" referred to mathematics knowledge, rules, formulas, axioms, and theorems, whereas the "basic skills" included the capabilities of children to perform computations, simple reasoning, and data processing, and draw tables and figures [16]. It was widely believed that this emphasis on the "two basics" helped Chinese students to build a solid foundation in mathematics [17].

The dissemination of the Preschool Education Outline (1981) weakened subject boundaries [14]. This process continued in 2001 when the Chinese government published the Preschool Education Guidelines (also called the New Outline), which took Chinese early mathematics education into a new stage [12]. The New Outline divided early childhood education into five domains: health, language, social studies, science, and art [18]. The content in the mathematics curriculum was incorporated into the domain of science. The "two basics" that had been proposed in the 1980s remained the key elements of early mathematics, but the New Outline specified the key concepts that should be covered. These concepts included "Number", "Number Operation", "Space", "Shape", "Set", "Data Analysis", "Time", and "Pattern." Preschool teachers were required to design appropriate activities according to children's developmental stages, interests, and real-life experiences [19].

In 2012, the Guideline for the Development of Children Aged 3–6 (the Guideline) was issued, which explicitly specified expectations for children's mathematical learning and development at different ages [20]. Compared with the previous policies, both the New Outline and the Guideline required early mathematics learning and teaching activities to shift from the traditional teacher-led mode to a child-centered approach that took full advantage of children's daily experiences and helped them to identify, analyze, and solve real-life problems [18,20]. Nevertheless, mathematics-specific activities, in which mathematics was taught as a separate subject to young children, were still considered to be important in Chinese preschools [21]. Although there were age-specific mathematics learning requirements, contents related to "Number" and "Number Operation" remained the primary focus of preschool mathematics teaching [22].

In terms of pedagogy, collective teaching tends to be the most popular teaching mode in early mathematics activities for young Chinese children [23]. Manipulable materials and stories are usually used as teaching aids to enhance children's engagement [19]. In particular, the mathematics activities in Chinese preschools are usually conducted over a period of 45 to 50 min and have a standardized structure: they start with the teacher's introduction to the topic, which is followed by the children's participation in one to three relevant games or activities, and they end with the children reporting back and an explicit summary from the teacher [24,25]. Children are provided with various opportunities for practice and manipulation during the standardized teaching activities and are expected to master the "two basics" [17]. The high level of standardization in activity organization leads to a high level of teacher-centeredness compared with the teaching of mathematics in Western countries [17].

1.2. Chinese Students' Mathematics and Problem-Solving Performance

Problem-solving competence is defined as "an individual's capacity to engage in cognitive processing to understand and resolve problem situations where a method of solution is not immediately obvious" [8]. A problem-solving assessment has been integrated into the PISA test since 2003 and has been treated as a core component since PISA 2012. The problem-solving assessment is focused on students' general reasoning skills, their ability to regulate problem-solving processes, and their willingness to solve problems, and it does so by confronting students with problems that do not require expert knowledge to solve [8]. Although subject knowledge is not highlighted in the problem-solving assessment, most of the subject-related content included in the problem-solving component of PISA involves mathematics.

Researchers have typically classified problems into routine problems and non-routine problems [26,27]. Routine problems require the problem solver to apply standardized procedures to come up with fixed solutions, while non-routine problems require tacit knowledge, which is only imperfectly described in terms of a set of rules [8]. Flexibility is required to solve non-routine problems [28]. Such flexibility requires increased higher-order thinking and logical reasoning from children as they apply their knowledge [29,30]. Non-routine problems appear frequently in the PISA problem-solving component [8]. For example, students may be asked to determine the quickest route linking two places based on the travel time provided for each part of various different possible routes. Students can only solve the problem by identifying multiple potential routes, calculating the length of time for each route, and comparing the results.

Chinese students' outstanding performance in mathematics evaluations, such as PISA, is usually considered to be associated with their mathematics learning experiences in the early years, especially the emphasis on the "two basics" [2]. However, there is little research investigating this assumption. Similarly, it is not clear whether Chinese students' relative underperformance in problem solving compared with the performance of other students around the world with similar overall scores in mathematics can be traced back to their early mathematics learning experiences.

The cohorts of secondary students who participated in PISA 2012 and 2015 were born soon after the New Outline and the Guideline were issued. They were the first generation to benefit from the curricular changes prescribed in these documents. Although it is difficult to determine the early mathematics learning experiences of these students, a content analysis of contemporary teaching reference books can provide insights into whether and how mathematical problem-solving skills are facilitated in the early years in Chinese preschools. However, it is important to remember that the emphasis on child-centered learning and the problem-solving component in current early mathematics activities is much greater now than it was 20 years ago.

1.3. Content Analysis in Mathematics Education Research

Content analysis, which uses systematic coding to compile, compare, and evaluate the content in texts, is frequently used when synthesizing learning content [10]. For example, TIMSS analyzed hundreds of mathematics textbooks and other curricular materials from 50 countries to compare mathematics education in these countries [31]. The key concepts in mathematics textbooks for primary and secondary schools have also been examined to reveal how mathematics topics are presented at different stages of primary education [32], how different pedagogies are used in the delivery of this mathematics content [32,33], and how problem solving is facilitated in mathematics education [34].

Chinese preschool teachers typically use teaching reference books to help them arrange their daily teaching activities. Weekly, monthly, and yearly teaching plans are clearly outlined in the books for easy reference for preschool teachers [35]. Chinese preschool teachers are highly dependent on these teaching reference books, as the books usually serve as the main resources for activity design. In a study by Yang [13], it was found that 78% of the participating preschool teachers carefully followed the arrangements suggested in the reference books.

Against this background, the current study examined the content covered in early mathematics education and its suggested delivery in Chinese preschools through a content analysis of eight commonly used sets of preschool teaching reference books. We aimed to portrait an overall picture of how mathematics education is implemented in Chinese preschools and to provide insights into how to better support children's mathematics performance, as well as the cognitive processes that facilitate the application of mathematics knowledge in the early years. In particular, we examined (i) the teaching approaches recommended in the suggested mathematics activities, (ii) the mathematical concepts covered in these activities, and (iii) whether a problem-solving component was integrated into the activities.

2. Methods

2.1. Selection of Preschool Teaching Reference Books

To ensure the representativeness and quality of the teaching reference books to be included in the analyses, we invited 35 experienced preschool teachers, who were either recommended by researchers of early mathematics education or identified from the professional networks of the authors, to make initial recommendations on the best teaching reference books that they have ever used. These 35 preschool teachers were from 10 provinces (across 34 provinces in total) in China and at least two of them were from the same province. The second author briefed the purposes of this study to these teachers individually on WeChat, the foremost social platform in China, and made further elaborations if needed to ensure that all the teachers were clear about the requests. All 35 preschool teachers agreed to participate in this study. Fourteen nominations were received. We then looked up the sales volumes for these 14 teaching reference books on jd.com and dangdang.com, the two largest book-selling websites in China, to determine their level of popularity. Only books with a top-10 sales volume in the category of "early childhood education teaching reference books" remained on the list.

To ensure that the suggested teaching activities were of high quality, we included only those books edited by top scholars or experienced preschool teachers and published by university presses with a strong reputation. Among the 10 sets of nominated teaching reference books that remained after the initial screening, eight were edited by either well-known scholars from universities or by the local education departments. Two were edited by individual editors who were not from either the university or government sectors. We searched for the profiles of these editors but found that they held no professional qualifications on early childhood education. Therefore, we only retained the eight sets of teaching reference books (containing a total of 58 books) for the content analysis that had been edited by either scholars from universities or by the local education departments. The details of these sets, referred to as A to H in the analyses below, can be found in Table 1. These reference books provide systematic semesterly, monthly, and weekly teaching plans and detailed activity designs for preschool teachers to follow. Teachers select the appropriate academic year when teaching children aged 3 to 6 years (the lower, middle, and upper classes).

Table 1. Kindergarten teachers' teaching reference books analyzed in this study.

	Teaching Reference Books	Editor(s)	Publisher	Year of Publication	Ranking on dangdang.com as of July 2020	Ranking n jd.com as of July 2020
A	Learning Activity	Zhu, Jiaxiong	Shanghai Education Press	2009	1	1
B	Happy and Developmental Curriculum in Kindergarten	Luo, Jie	Beijing Normal University Publishing Group	2010	2	2
C	Integrate Activity Curriculum in Kindergarten	Zhou, Jing; Zhang, Xinru	Nanjing Normal University Press	2014	4	3
D	Learning and Development through Experience Curriculum in Kindergarten	Bian, Xia; Wang, Jinqiu; Huang, Jin; Zhang, Jinmei	Zheijing Education Publishing House	2015	6	7
E	Kindergarten Curriculum Guidance in Zhejiang Province	Qin, Jinliang	Zhejiang Gongshang University Press	2018	3	4
F	Kindergarten Curriculum Guidance in Shandong Province	Yu, Yongping; Fang, Ming;	Tomorrow Publishing House	2018	7	5
G	Kindergarten Teacher Reference Book in Fujian Province	Lin Jing, Chen Yafang, Zheng Jiancheng, Lin Xiujuan	Fujian People's Publishing House	2017	8	8
H	Penetrative Learning Curriculum in Kindergarten	Zhao Jishi, Tang shu	Nanjing Normal University Press	2017	5	6

2.2. The Content Analysis Coding Framework

Quality methodologies are crucial for understanding the contents covered using content analysis. As suggested by Polikoff et al., content analysis with unsystematic methodologies will fail to provide useful insights into the contents of books [36]. To ensure that the results of the current content analysis were able to offer an in-depth understanding of the three aspects of early mathematics teaching in Chinese preschools, we developed a systematic coding framework using the suggested learning activities as the unit of analysis. Three coding dimensions were developed to reflect (i) the teaching approaches adopted in the delivery of mathematics activities, (ii) the mathematical concepts covered in the activities, and (iii) whether or not the mathematical concepts were taught in a problem-solving scenario. As straightforward techniques involving counts are considered to be good approaches to textbook analyses [36], the codes developed in the coding framework in this study allowed us to easily identify whether the descriptors of each coding dimension appeared in the mathematics activities or not.

Teaching approaches adopted in the mathematics activities. Teaching approaches were coded according to whether the activity was conducted in a collective teaching mode or a learning corner teaching mode, both of which are suggested in the books. Collective teaching activities were operationally defined as activities in which teachers organize the teaching activities as part of their whole-class instruction, while learning corner teaching activities were defined as activities in which teachers provide mathematical manipulations for children to explore individually in the mathematics learning corner. We also considered whether the collective teaching activity was a mathematics-integrated activity, in which mathematics was one of the multiple learning areas, or a subject-specific activity, in which mathematics was the only learning area covered. Finally, we coded whether the activity adopted the above-mentioned standardized teaching procedure, whereby: (1) the teacher introduces the topic, (2) the children participate in one to three games or activities on the topic, and (3) the children report back and the teacher delivers an explicit summary.

Mathematical concepts covered in the activities. Following the Guideline [20], mathematical concepts were categorized into "Number", "Number Operation", "Measurement", "Space", "Shape", "Sets", "Time", "Data Analysis", and "Pattern." For example, the activity "I Am Growing Up" was coded as a mathematics-integrated activity, in which the key mathematical concept, "Measurement", was integrated into a theme with which children were very familiar and in which children's language and social skills were also highlighted as key learning objectives. In contrast, the activity "Motor Show" was a typical mathematics-specific activity. In this activity, the mathematical concept of a "Set" was taught via an activity that required children to sort toys in different ways. Children were expected to understand the mathematical concept of a "Set" through multiple trials. Mastery of the key mathematics concept "Set" was the only learning objective of this activity; therefore "Motor Show" was coded as a mathematics-specific activity and as relating to the concept of a "Set."

Whether or not the mathematical concepts were taught in a problem-solving scenario. Finally, the identified mathematics activities were coded as "problem-solving activities" or "non-problem-solving activities" according to whether problem-solving scenarios were integrated into the activities. As both the New Outline and the Guideline emphasize the importance of real-life problem solving in early mathematics learning and teaching, problem-solving activities were further coded as activities whereby children solve real-life problems or activities where they solve problems not relevant to their daily lives.

We also coded mathematical problem-solving activities as routine or non-routine problem-solving activities based on whether there was flexibility in the problem-solving strategies or solutions. For example, the "Parking" activity was coded as a routine activity, in which it was suggested that teachers provide "cars" of several sizes for children to park in the correct parking space according to their size. Each car had only one suitable parking space, so the solution was fixed. The "Using CNY 10 to Buy Vegetables" activity was coded as a non-routine activity. In this activity, it was suggested that teachers give CNY 10 to children and ask them to buy vegetables for dinner in a shopping mall. To solve this problem, children were allowed to have different combinations in their purchase plan as long as the total amount for the purchase was CNY 10 or less.

2.3. Procedure

All of the learning activities presented in the teaching reference books were first reviewed to determine whether they involved mathematical learning. Only activities with mathematical components were included in the content analysis. The second author reviewed all of the activities suggested in the eight sets of teaching reference books and completed the coding process for all mathematics-related learning activities. One undergraduate student majoring in early childhood education also coded 10% of the mathematics activities, which were randomly selected from the overall pool of activities. An agreement of 96.4% was achieved between the two coders after two rounds of checking and discussion. Discrepancies between the two coders were resolved through discussion, in consultation with an experienced frontline teacher, who was one of the 35 experienced preschool teachers nominating the quality teaching reference books at the initial stage of this study, and who had specialization in early mathematics education, until 100% agreement was achieved.

3. Results

3.1. Teaching Approaches Adopted in the Mathematics Activities

We identified 2038 mathematics activities from a total of 11,024 activities suggested in the eight sets of teaching reference books. As shown in Table 2, for around 70% of the mathematics activities, the suggested delivery mode was collective teaching (n = 1419). The remaining 30% were to be conducted in learning corners (n = 619). Most of the collective mathematics activities (69.3%, n = 984) were integrated into activities that aimed to support children's development in multiple domains, with only around one third of the collective mathematics activities being mathematics specific (n = 435). The frequency of mathematics activities was higher in the reference books for older children compared with those for younger ones (χ^2 (df = 2, n = 2038) = 265.798, $p < 0.001$; n = 601, 651, and 786, for lower, middle, and upper classes, respectively), as was the number of collective mathematics activities (χ^2 (df = 2, n = 1419) = 167.138, $p < 0.001$; n = 381, 466, and 572, for lower, middle, and upper classes, respectively). The frequency of learning corner mathematics activities was similar for each age group (χ^2 (df = 2, n = 619) = 100.16, $p < 0.001$; n = 220, 185, 214, for lower, middle, and upper classes, respectively).

We found that all of the collective mathematics activities adopted the three-phase standardized procedure mentioned above. In contrast, the role of teachers was rarely mentioned in the learning corner activities, with the only suggestion for teachers typically being to provide specific mathematics materials or resources for children's manipulation.

Table 2. The distribution of different mathematics activities covered in kindergarten teachers' teaching reference books.

	A	B	C	D	E	F	G	H	Total
Total number of activities suggested	538	1784	1149	1334	1137	1632	1907	1543	11,024
Total number of mathematics activities suggested	145	296	276	489	177	177	301	177	2038
Suggested teaching approaches									
Collective activities	145	162	235	282	177	73	228	117	1419
Integrated activities	145	162	86	86	177	73	138	117	987
Mathematics-specific activities	0	0	149	196	0	0	90	0	435
Learning corner activities	0	134	41	207	0	104	73	60	619
Mathematical concepts covered									
Measurement	20	45	32	54	16	14	30	24	235
Number	37	103	59	181	42	48	76	53	599
Number Operation	22	16	19	66	12	29	25	30	219
Space	6	16	16	24	9	4	12	9	96
Shape	12	24	28	43	15	25	34	17	198
Data Analysis	19	29	17	31	27	11	39	8	181
Set	37	57	59	73	53	25	70	26	400
Pattern	5	18	35	36	18	10	16	9	147
Time	8	19	9	15	9	3	16	3	82
Problem-solving oriented or not									
No problem-solving components	101	214	225	399	138	154	220	135	1586
Integrated problem solving	44	82	51	90	39	23	81	42	452
Solve real-life problems	29	55	29	90	39	23	81	42	295
Solve non-real-life problems	15	27	22	59	31	15	56	21	153
Solve routine problems	27	46	34	60	19	15	55	31	287
Solve non-routine problems	17	36	17	30	20	8	26	11	165

3.2. Mathematical Concepts Covered

Figure 1 shows the distribution of various mathematical concepts in the suggested collective teaching and learning corner activities. The content most frequently covered related to "Number" (26.75% of the collective and 30.09% of the learning corner activities), "Set" (18.81% of the collective and 17.93% of the learning corner activities), "Measurement" (10.94% of the collective and 10.79% of the learning corner activities), and "Number Operation" (10.21% of the collective and 10.03% of the learning corner activities). In contrast, "Pattern" (5.60% of the collective teaching and 9.57% of the learning corner activities), "Space" (4.74% of the collective teaching and 3.80% of the learning corner activities), and "Time" (4.34% of the collective teaching and 2.58% of the learning corner activities) were covered less frequently.

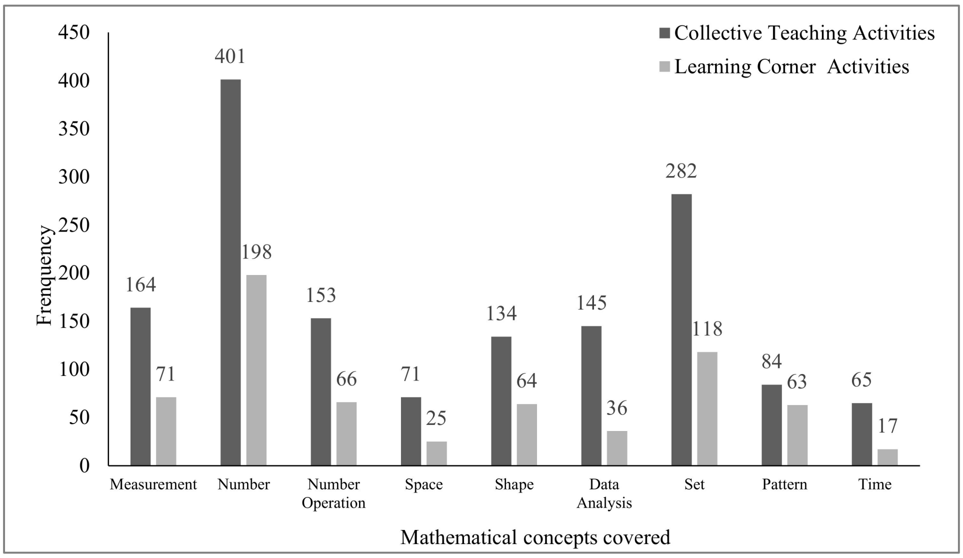

Figure 1. Distribution of different mathematical concepts covered in collective teaching and learning corner activities.

We further examined whether the distribution of mathematical concepts across the two types of mathematics activities varied according to age group (See Table 3). For the 5-year-old group, compared with the 3- and 4-year-old groups, we found a significant decrease in the percentage of "Number" activities over time (χ^2 (df = 2, n = 401) = 37.02, $p < 0.001$; χ^2 (df = 2, n = 198) = 17.30, $p < 0.001$, for collective and learning corner mathematics activities, respectively) and a sharp increase in the number of "Number Operation" activities (χ^2 (df = 2, n = 153) = 141.24, $p < 0.001$; χ^2 (df = 2, n = 66) = 58.24, $p < 0.001$, for collective and learning corner mathematics activities, respectively). A gradual decrease in the proportion of "Set" activities was found when comparing the activities for 5-year-olds with those for 3-year-olds in both collective teaching (χ^2 (df = 2, n = 282) = 7.72, $p < 0.05$) and learning corner activities (χ^2 (df = 2, n = 118) = 40.39, $p < 0.001$). There were some "Time" activities suggested for 3-year-olds, but the frequency decreased for 4-year-olds and increased again for 5-year-olds (χ^2 (df = 2, n = 82) = 66.61, $p < 0.001$; χ^2 (df = 2, n = 65) = 51.82, $p < 0.001$; χ^2 (df = 2, n = 17) = 4.77, $p < 0.05$, for 3-year-olds, 4-year-olds, and 5-year-olds, respectively). We did not find age-related differences in the percentages of "Measurement" and "Pattern" activities in either collective teaching (Measurement: χ^2 (df = 2, n = 164) = 2.06, p = 0.357; Pattern: χ^2 (df = 2, n = 84) = 3.07, p = 0.215) or learning corner activities (Measurement: χ^2 (df = 2, n = 71) = 1.72, p = 0.424; Pattern: χ^2 (df = 2, n = 63) = 1.81, p = 0.405).

Table 3. The distribution of different mathematical concepts across all age groups.

	Collective Mathematics Activity			Learning Corner Mathematics Activity		
	n	%	χ^2	*n*	%	χ^2
Measurement			2.06			1.72
3-year-old	55	33.54		28	39.44	
4-year-old	47	28.66		19	26.76	
5-year-old	62	37.80		24	33.80	
Number			37.02 ***			17.30 ***
3-year-old	128	31.92		71	35.86	
4-year-old	186	46.38		87	43.94	
5-year-old	87	21.70		40	20.20	
Number Operation			141.24 ***			58.24 ***
3-year-old	0	0.00		0	0.00	
4-year-old	3	1.96		2	3.03	
5-year-old	150	98.04		64	96.97	
Space			2.90			14.48 ***
3-year-old	26	36.62		17	68.00	
4-year-old	17	23.94		6	24.00	
5-year-old	28	39.44		2	8.00	
Shape			8.08 *			2.38
3-year-old	35	26.12		16	25.00	
4-year-old	60	44.78		26	40.60	
5-year-old	39	29.10		22	34.40	
Data Analysis			69.61 ***			2.78
3-year-old	2	1.38		0	0.0	
4-year-old	63	43.45		13	36.11	
5-year-old	80	55.17		23	63.89	
Set			7.72 *			40.39 ***
3-year-old	116	41.13		71	60.17	
4-year-old	83	29.43		30	25.42	
5-year-old	83	29.43		17	14.41	
Pattern			3.07			1.81
3-year-old	21	25.00		18	28.57	
4-year-old	34	40.48		26	41.27	
5-year-old	29	34.52		19	30.16	
Time			51.82 ***			4.77 *
3-year-old	9	13.85		4	23.53	
4-year-old	7	10.77		0	0.00	
5-year-old	49	75.38		13	76.47	

* $p < 0.05$, *** $p < 0.001$.

Most “Data Analysis” collective teaching activities were found for 4- and 5-year-olds, with only a few suggested for 3-year-olds (χ^2 (df = 2, n = 145) = 69.61, $p < 0.001$). A similar pattern was found for “Data Analysis” learning corner activities, but not to a statistically significant degree (χ^2 (df = 2, n = 36) = 2.78, p = 0.096). The frequency of “Space” activities was low for all age groups; however, there was a significant decrease in the number of suggested “Space” learning corner activities for older children (χ^2 (df = 2, n = 25) = 14.48, $p < 0.001$), but not in “Space” collective teaching activities (χ^2 (df = 2, n = 71) = 2.90, p = 0.234). The percentage of “Shape” activities was also low across all age groups, but there was a significant increase in the number of suggested collective “Shape” activities for 4-year-olds and a decline for 5-year-olds (χ^2 (df = 2, n = 134) = 8.08, $p < 0.05$). No significant differences were identified in the number of suggested learning corner “Shape” activities across different age groups (χ^2 (df = 2, n = 64) = 2.375, p = 0.31).

3.3. The Problem-Solving Component in Mathematics Activities

The reference books suggested that preschool teachers create real-life or story situations and provide manipulable materials to help children learn mathematical concepts in a playful way, but only 22% of the mathematics activities (n = 452) were coded as integrating a problem-solving component. The percentage of activities that integrated a real-life problem-solving component was even smaller (14%, n = 295). Over 70% of the problem-solving activities were collective teaching activities (n = 319). In addition, of the activities categorized as integrating a problem-solving component, over half of the mathematics problems were routine problems (63%, n = 287), while the frequency of non-routine problems was lower (37%, n = 165).

Although the overall percentage of problem-solving activities across all mathematics activities increased with the age of the children (χ^2 (df = 2, n = 452) = 68.53, $p < 0.001$), more real-life (χ^2 (df = 2, n = 295) = 34.58, $p < 0.001$) and non-routine (χ^2 (df = 2, n =165) = 35.83, $p < 0.001$) mathematics problem-solving activities were suggested for older than for younger children (see Table 4).

Table 4. The distribution of problem-solving related mathematics activities across all age groups.

	Problem-Solving Activities			Activities to Solve Real-Life Problems			Activities to Solve Non-Real-Life Problems			Activities to Solve Routine Problems			Activities to Solve Non-Routine Problems		
	n	%	χ^2	n	%	χ^2	n	%	χ^2	n	%	χ^2	n	%	χ^2
			68.53 ***			34.58 ***			35.83 ***			65.40 ***			9.22 ***
3-year-old	139	30.75		70	23.73		69	43.95		129	44.95		10	6.06	
4-year-old	138	30.53		97	32.88		41	26.11		75	26.13		63	38.18	
5-year-old	175	38.72		128	43.39		47	29.94		83	28.92		92	55.76	

*** $p < 0.001$.

3.4. Activities Suggested in Each Set of Teaching Reference Books

To ensure that the distribution of activities in each set of teaching reference books was in line with the distribution pattern identified in the overall pool of activities, we checked the number of different types of activities in each set of reference books. As shown in Table 2, the distribution within each set followed a similar pattern in terms of suggested teaching approaches, mathematical concepts covered, and the inclusion of a problem-solving component. We therefore considered that the results of the content analysis using this pool of activities were indicative of the general situation in terms of Chinese preschool teachers' implementation of early teaching and learning mathematics activities with the support of any of these popular teaching reference book sets.

4. Discussion

In this study, we conducted a content analysis of eight sets of commonly used preschool teaching reference books to understand the mathematical content that is taught in Chinese preschools and how it is delivered. The analysis was designed to provide insights into how to better support Chinese children's mathematics learning from the early years onwards, given that Chinese secondary students generally perform well in mathematics but score relatively low on mathematics-related problem-solving assessments.

4.1. The Delivery of Mathematics Content

The results showed that Chinese preschool teachers were guided by the teaching reference books to consolidate children's mastery of basic mathematical concepts and skills, primarily through collective teaching activities. The frequency of suggested collective mathematics activities was higher for older than for younger children. This finding provides further evidence of the important role of collective teaching in Chinese early childhood education [19,37].

In addition, the common teaching steps, as indicated in the literature—teacher introduction, children's active participation, and children reporting back followed by teacher summary [25]—were clearly identifiable in the design of the suggested collective teaching activities. Such well-designed activities can reduce ambiguity and confusion for the teachers who implement them and aid children as they progress toward their learning goals of memorizing and understanding mathematical concepts and skills [17]. The activities are also effective at delivering teaching content to relatively large classes (over 30 children). Playful practice and exploration opportunities were usually embedded in the second phase of teaching to ensure children's mastery, indicating a move from explicit teaching to assisted discovery in early mathematics teaching [2]. The integration of multiple play-based and exploration components in this standardized tripartite teaching procedure also reflected an emphasis on children's construction rather than teachers' instruction in the learning process. Therefore, a fusion of "Eastern" and "Western" pedagogies in early mathematics learning is evident in the suggested mathematics learning activities, as has been identified in other learning areas in Chinese preschools [2,38].

Learning corner activities were one of the two suggested approaches to early mathematics learning in these teaching reference books (n = 619, 30% of mathematics-related activities). However, practical suggestions related to teacher support for mathematics learning in the learning corner activities were rare in the books. Teachers were usually advised to provide materials for children to manipulate in learning corners as an extension of collective teaching. Learning corners are important for supporting children's application and practice of the concepts or skills taught by teachers in formal teaching activities, and teacher scaffolding is crucial when children play in the learning corners [39]. Learning corners can be perfect venues for children to practice solving real problems using the mathematical concepts that they have learned in collective teaching activities, but only if teachers are able to establish appropriate scenarios and provide the necessary materials. Close observations of children's participation in the activities and teacher–child interactions while children are playing with the materials in the learning corners are also helpful for teachers when they are identifying possible learning difficulties [40]. Given the guiding role that teaching reference books play in preschool education in China, it is reasonable to infer that the value of learning corners has not been sufficiently recognized in Chinese preschools.

4.2. The Mathematics Content Covered

Our results echoed the long-standing emphasis on the "two basics" in mathematics teaching and learning in the early years [17]. Number-related concepts, such as "Number", "Number Operation", and "Measurement", and concepts related to children's general thinking capacities, such as "Set", were covered in detail, suggesting a heavy emphasis on these concepts in the early years.

Key mathematical concepts were systematically arranged for children at different age levels, reflecting their learning progression as they master these concepts. For example, although number-related activities were highlighted for children of different ages, younger children were provided with more activities on "Number", while more activities related to "Number Operation" and "Data Analysis" were suggested for older children. Fewer activities related to fundamental mathematical concepts in the areas of "Shape" and "Set" were suggested for older children. This is in line with previous findings that have indicated that Chinese preschool teachers typically teach age-specific content, such as that provided in the teaching reference books [2]. Indeed, a logical sequence when teaching different mathematical concepts is essential for proficient mastery of the "two basics" in the early years, which in turn lays a solid foundation for more sophisticated mathematics learning at the primary and secondary stages.

4.3. Early Exposure to Mathematics Problem Solving

The content analysis clearly identified the common practice of emphasizing the "two basics" in the early years. This approach, together with the continuation of such practices during primary and secondary mathematics education in China, is believed to contribute to Chinese students' excellent mathematics performance [33]. However, we are concerned about Chinese students' lack of proficiency in problem solving, as shown in the PISA assessments. In the content analysis, we found that only around one fifth of the suggested mathematics learning activities integrated a problem-solving element, with greater emphasis placed on real-life and routine problem-solving activities than on non-real-life and non-routine problem-solving activities.

Since the launch of the New Outline in 2001, it has been suggested that teachers support children's mathematics learning not only in terms of mathematical knowledge, but also by applying the mathematical knowledge to real-life scenarios [18]. In the content analysis of the teaching reference books, we found that a low percentage of activities prompt children to learn to solve either real-life or non-real-life problems. These problems, when they appear, tend to be similar to the "application exercises", which are popular in primary and secondary mathematics education: they more consistently use routine problems, with explicit information on the problems, the necessary clues provided, and the particular solutions expected. In addition, the problem-solving scenarios in these suggested activities are static, with all of the relevant information for solving the problem disclosed at the outset, rather than interactive, in which not all information is disclosed and some information has to be uncovered by exploring the problem situation. Interactive problems are more common in our daily lives [8].

Although these findings are not able to fully explain Chinese students' relative lack of proficiency in problem solving, they suggest that the teaching of mathematical problem solving is not a prominent activity in current mathematics education in Chinese preschools [41]. This may lead to a lack of proficiency in early mathematical problem solving. Since the launch of the Guideline in 2012, problem-solving abilities have been considered to be highly important in early childhood education and Chinese preschool teachers are found to highly acknowledge the role of problem solving in the early years [25]. However, little is known about preschool teachers' understanding and implementation of early mathematical problem solving. Therefore, increased effort should be invested in integrating a problem-solving component into early mathematics education.

In summary, the results of the content analysis of the eight sets of popularly used and well-recognized teaching reference books revealed the emphasis on the "two basics" and teachers' role in leading the collective mathematics teaching in early mathematics activities. However, the results also revealed the limitations in the design of these activities in terms of a lack of either teachers' guidance in learning corner mathematics activities or the integration of a problem-solving component, especially a real-life and non-routine problem-solving component. It is highly important for frontline preschool teachers to be aware of such features in the teaching reference books, given their high degree of loyalty to adopting the teaching suggestions in their teaching practices [12]. We strongly suggest that preschool teachers carefully select and review the activities in the teaching reference books before implementation instead of treating the teaching reference books as an absolute authority. It is also helpful to form learning communities within and across different preschools to work together when selecting appropriate teaching contents from the teaching reference books and in enriching and adapting the selected activities to better support children's mathematics learning [42].

4.4. Limitations and Future Directions

This study demonstrates an innovative and cost-effective way to understand early mathematics education in Chinese preschools. The findings have implications in terms of improved support for students' mathematics learning outcomes in China. However, limitations in the research design, data availability, and data analysis may have influenced the accuracy of the findings in relation to the actual implementation of early mathematics education in practice.

First, our findings are based on an analysis of eight sets of commonly used teaching reference books. Although these teaching reference books are popular compared with others on the market, we do not have exact data on how these teaching reference books are used by preschool teachers. Given that existing studies have shown that Chinese preschool teachers typically demonstrate high fidelity to teaching reference books in their curriculum design and teaching activities, we believe that the findings of the content analysis are reliable and present an overall snapshot of the implementation of early mathematics education in Chinese preschools. Future studies, however, should include classroom observations and teacher interviews to provide more in-depth information on how early mathematics education is perceived and implemented in Chinese preschools.

Second, the students who participated in the PISA assessments and whose performance was mentioned in this study were not the same generation whose teachers used the teaching reference books under analysis. Based on our knowledge of the evolution of early mathematics education in China, we believe that the early mathematics learning experiences of the PISA participants were more knowledge centered and teacher directed than those of current preschool students. It is therefore important to obtain longitudinal data and link children's early learning experiences with their later mathematics performance, especially on problem-solving-related items, to draw more accurate conclusions about how students' mathematics learning can be better supported in the early years.

Finally, our findings are only based on Chinese children's mathematics learning experiences in preschool. Children's performance is shaped by all of their learning experiences, both inside and outside of school. Therefore, evidence relating to children's mathematics learning experiences at different stages (pre-primary, primary, and secondary) and in various contexts (school and family) is necessary to provide a more coherent and comprehensive picture of Chinese students' mathematics learning experiences. Nevertheless, considering the universal influences of Chinese culture on education and the continuity of the education systems in China, the findings from this study still provide important insights into how to better support children's early mathematics learning. Moreover, the results strongly suggested conducting further systematic studies into mathematics education at both primary and secondary levels, especially with regard to the teachers' role in students' learning, as well as the facilitation of mathematical problem-solving capacities at different stages, to better understand the philosophies and pedagogies of mathematics education at different school levels, as practices at the pre-primary stage might continue throughout the rest of a child's formal education.

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