A Bioeconomic Analysis of a Renewable Resource in the Presence of Illegal, Unreported and Unregulated Fishing

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Original Research Article

Abstract

The issue of illegal, unreported and unregulated (IUU) fishing is of prime concern to fisheries in developing countries where the regulatory regimes are often weak. This study proposes a Gordon-Schaefer bioeconomic model with non-constant catchability and nonlinear cost to study the impact of IUU fishing on the stock size of a marine fishery in Ghana. The static equilibrium reference points of the model are established and discussed. Bifurcation analysis on the modified Schaefer model shows the existence of a transcritical bifurcation point were the model is structurally unstable. Pontryagin's maximum principle is employed to investigate the necessary conditions of the model, and also established are the sufficiency conditions that guarantee the existence and uniqueness of the optimality system. The characterization of the optimal control gives rise to both the boundary and interior solutions, with the former indicating that the resource should be harvested if and only if the marginal revenue of harvest exceeds the marginal revenue of stock. Numerical simulations with empirical data on the Ghana sardinella are carried out to validate the theoretical results. It is shown that IUU fishing leads to excessive exploitation of the resource biomass to levels below 50% of the carrying capacity. This has the tendency of making the fishery unsustainable, with its concomitant loss of revenue to fishers.

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1 Introduction

The marine fishing industry plays a vital socioeconomic role in Ghana and comprises three sectors, namely artisanal, semi-industrial and industrial. Among these sectors, the artisanal sector is the most prominent in terms of the number of fishers and the quantum of catches. The artisanal fishery sector is open access with minimal or nonexistent regulatory framework. This open access nature of the fishery breeds unhealthy competition among the fishermen, with the mentality of each fisherman being, "If I don't catch the fish, someone else will or might!" [1]. It is therefore not surprising that faced with dwindling fish stocks, individual fishermen would resort to extreme measures to outperform the competition. These measures include the use of under-sized mesh gears, use of light (or attractants) in fishing, use of explosives like dynamites, and use of chemicals.

To buttress this fact, Koranteng [2] says it is common practice for fishermen operating in an over-exploited and poorly managed fishery to use smaller-sized meshes as a means to increase their catch. Pauly et al. [3] and Pauly [4] describe such a scenario as *Malthusian overfishing*, which is a situation where "poor fishers, faced with declining catches and lacking any other alternative, initiate wholesale resource destruction in their effort to maintain their incomes". Pauly et al. give the symptoms of Malthusian overfishing as: (i) use of gears and mesh sizes that are not sanctioned by government, (ii) use of gears not sanctioned within the fisher-folk communities, (iii) use of gears that destroy the resource base, and (iv) use of 'gears' such as dynamite or sodium cyanide that compromise the marine habitat as well as endangering the fisher-folks themselves [2, 5].

The few Government regulations on the artisanal fishery sector include adherence to a minimum mesh size of 25 millimeters, approximately one inch, in stretched diagonal length. This regulation has been wantonly disregarded by the fishermen, rendering it ineffective. The main argument advanced by the fishermen is that the minimum size of 25 millimeters in stretched diagonal length cannot catch some of the targeted species like anchovies. Thus, they continue to use unapproved mesh sizes to disastrous consequences for the artisanal fishery sector [6].

The assumption of constant catchability coefficient in the standard Gordon-Schaefer bioeconomic model [7, 8] is at variance with technological aspects of the evolution of fishing power. Improvement in the design of fishing gear, such as the use of synthetic fibers coupled with technologies for fish detection, leads to increased precision in the application of fishing power, which is rarely explicitly considered when standardizing fishing effort [9, 10, 11]. Furthermore, while the effects of climatic conditions induced by global warming on the decline of the sardinella fishery could not be discounted, excessive exploitation by fishermen through the use of fishing gear with unapproved mesh sizes and also the use of attractants exacerbates the problem. To investigate this claim, sensitivity analysis is performed on the catchability parameter in the proposed model.

There is a dearth of literature on modeling the effects of illegal, unreported and unregulated (IUU) fishing on marine ecosystems of developing countries. Petrossian [12] contends that IUU fishing negatively impacts the ecosystem and poses a threat to the livelihoods of those who depend on it for their sustenance. Fishers indulging in IUU fishing is the main contributor to overexploitation of marine species, and also hinders the recovery of the biomass and ecosystems [13]. Therefore, a bioeconomic model with a quadratic cost function of fishing effort is developed to assess the possible impact of the use of these IUU fishing practices on specifically the catchability of the sardinella. This

quadratic cost function is a modification of the linear cost function usually employed in resource modeling. See, for example, studies conducted by Kar and Misra [14] and Dubey et al. [15]. Also novel to this present work is the discussion of the relationship between the shadow price and the net revenue per unit harvest as it relates to the optimal fishing effort. Formulation of the optimal control model comprising the biomass dynamics as well as the full bioeconomic model is outlined in Section 2. In Section 3, the dynamical properties of the model are explored through bifurcation analysis. In addition to characterizing the optimal control, the existence and uniqueness of the optimality system are portrayed in Section 4. Simulations employing empirical data on the Ghana sardinella fishery are presented in Section 5 while the summary and conclusions are discussed in Section 6.

2 Formulation of Model

The formulation of model takes into account the biological considerations as well as the economic objectives of fisheries management. Firstly, the biological dynamics are modelled taking cognizance of the impact of IUU fishing practices on the catchability of the fishery. Secondly, the complete bioeconomic model is formulated incorporating a quadratic cost function of the fishing effort, as opposed to the usual linear cost function.

2.1 The biomass dynamics

The catchability coefficient can be thought of conceptually as the probability of any single fish being caught. Catchability is also called fishing power, or sometimes gear efficiency [16]. Sensitivity analysis on the catchability coefficient of the model is performed to simulate the effects of IUU fishing on the fish stock.

The constant catchability coefficient q in the Schaefer model is replaced by $q(1 + \theta)$, where θ ($0 \leq \theta \leq 1$) is a proportion of the catchability and measures the effect of IUU fishing – under-sized mesh gears, light fishing, explosives and chemicals – on the level of biomass. Thus, the values of θ range from 0 (no IUU fishing) to 1 (extreme IUU fishing).

Therefore, the biological dynamics of the proposed model, also referred to as the modified Schaefer model, can be formulated as

$$\frac{dx(t)}{dt} = rx(t)\left(1 - \frac{x(t)}{K}\right) - q(1 + \theta)E(t)x(t)\,, \qquad x(0) = x_0, \tag{2.1}$$

where $x(t)$ is the biomass of the fish population (or stock size) at time t, x_0 is the initial stock size and r is the intrinsic growth rate of fish stock. In addition, $E(t)$ is the time-dependent rate of fishing effort while K represents the carrying capacity for the ecosystem [17, 18].

Note that the harvest or yield is given by

$$h^{\theta}(t) = q(1 + \theta)E(t)x(t). \tag{2.2}$$

There are two equilibrium points associated with Equation (2.1); namely, 0 and a positive equilibrium point

$$x_{eqm} = K\left(1 - \frac{q(1 + \theta)E}{r}\right), \tag{2.3}$$

provided that $E < \dfrac{r}{q(1 + \theta)}$.

When $E \geq \dfrac{r}{q(1+\theta)}$, $x_{eqm} \leq 0$ and the population goes into extinction. Therefore, $E = \dfrac{r}{q(1+\theta)}$ is a transcritical bifurcation point for the modified model. Thus the bifurcation point decreases with increasing values of θ. It is worth noting that the bifurcation point of the standard Schaefer model (where $\theta = 0$) is $E = \dfrac{r}{q}$.

The maximum sustainable yield (MSY) corresponds to the level of harvesting that maximizes the sustainable yield. That is, in theory, the maximum harvest which can be maintained indefinitely.

Substituting Equation (2.3) into Equation (2.2) gives the sustainable yield

$$h_S^\theta = q(1+\theta)EK\left(1 - \frac{q(1+\theta)E}{r}\right). \qquad (2.4)$$

The effort that maximizes the sustainable yield h_S^θ is found as

$$E_{MSY}^\theta = \frac{r}{2q(1+\theta)}. \qquad (2.5)$$

The value of MSY, denoted h_{MSY}^θ, is found by plugging Equation (2.5) into Equation (2.4). Hence,

$$h_{MSY}^\theta = \frac{rK}{4} \qquad (2.6)$$

and the biomass level at the MSY is

$$x_{MSY}^\theta = \frac{K}{2}.$$

It must be noted that when $\theta = 0$, E_{MSY}^θ reduces to the canonical Schaefer model reference point E_{MSY}. Of course, h_{MSY}^θ and x_{MSY}^θ are exactly the corresponding reference points h_{MSY} and x_{MSY}, respectively. For further details, see Ibrahim [19].

Consequently,

$$E_{MSY}^\theta = \frac{r}{2q(1+\theta)} = \frac{1}{1+\theta}E_{MSY}. \qquad (2.7)$$

Thus, as noted in Equation (2.2), the yield will now become

$$
\begin{aligned}
h^\theta &= q(1+\theta)Ex \\
&= qEx(1+\theta) \\
&= h(1+\theta).
\end{aligned}
$$

This implies that, to ensure sustainability of the resource (attaining equilibrium) at the MSY level when the catchability is increased by θ, the accompanying effort level E_{MSY}^θ should be reduced to a value that is $\dfrac{1}{1+\theta}$ of the E_{MSY}. Otherwise, the harvests h^θ would be a value $h\theta$ greater than the harvests without any increase in catchability, h. Furthermore, x^θ would be less than x_{MSY}. For more information on a modified catchability, see Mackinson et al. [20].

2.2 The bioeconomic model

Incorporating economic parameters into the afore-mentioned biological model gives the static Gordon-Schaefer bioeconomic model. The net revenue is the difference of total sustainable revenue TR_S and total cost TC, where TC is taken to be a quadratic function of E. That is,

$$TC = c_1 E + \frac{c_2}{2}E^2,$$

where c_1 and c_2 are the cost components relating to the effort. As stated by Hanson and Ryan [21], the additional quadratic cost term $\frac{c_2}{2}E^2$ may be seen as a perturbation on the usual linear cost, c_1E. It may also be viewed as a technique to avoid complexities inherent in the characterization of singular controls. The assumption is that both c_1 and c_2 are strictly positive, thereby making the costs to be monotonically increasing and growing rapidly than the corresponding linear costs (see Figure 1). Clark and Munro [22] as well as Sancho and Mitchel [23] asserted that quadratic costs are more in tune with reality than linear costs. Furthermore, the employment of a quadratic costs leads to the derivation of an explicit optimal control [24]. This is in contrast to linear costs, which give rise to bang-bang or singular controls [19].

Operating under an open-access regime where there is little or no regulation of the resource, effort E tends to a level where the sustainable economic rent (or net revenue) π_S is zero. This gives rise to what is known as open access yield (OAY). It must be noted that the OAY is also known as bionomic equilibrium (BE). The sustainable net revenue is given by

$$
\begin{aligned}
\pi_S &= ph_S^\theta - c_1E - \frac{c_2}{2}E^2 \\
&= pq(1+\theta)EK\left(1 - \frac{q(1+\theta)E}{r}\right) - c_1E - \frac{c_2}{2}E^2,
\end{aligned}
\tag{2.8}
$$

where p is the price per unit harvest.

Setting Equation (2.8) to zero gives

$$
E_{OAY}^\theta = \frac{2r\left[pq(1+\theta)K - c_1\right]}{2pq^2(1+\theta)^2K + rc_2},
\tag{2.9}
$$

where $pq(1+\theta)K > c_1$. Note that E_{OAY}^θ reduces to the standard Gordon-Schaefer reference point E_{OAY} when $\theta = 0$ and $c_2 = 0$.

The biomass level x_{OAY}^θ associated with E_{OAY}^θ is given by

$$
x_{OAY}^\theta = K\left(1 - \frac{q(1+\theta)E_{OAY}^\theta}{r}\right).
\tag{2.10}
$$

The corresponding harvest level is

$$
h_{OAY}^\theta = q(1+\theta)E_{OAY}^\theta x_{OAY}^\theta.
\tag{2.11}
$$

The level of harvesting that maximizes the sustainable net revenue is known as the maximum economic yield (MEY). The effort level that maximizes the net revenue is found from Equation (2.8) as

$$
E_{MEY}^\theta = \frac{r\left[pq(1+\theta)K - c_1\right]}{2pq^2(1+\theta)^2K + rc_2}.
\tag{2.12}
$$

Also, E_{MEY}^θ reduces to the Gordon-Schaefer reference point E_{MEY} when $\theta = 0$ and $c_2 = 0$, as well as being 50% of E_{OAY}^θ.

The associated biomass level is

$$
x_{MEY}^\theta = K\left(1 - \frac{q(1+\theta)E_{MEY}^\theta}{r}\right),
\tag{2.13}
$$

and the corresponding harvest level is

$$
h_{MEY}^\theta = q(1+\theta)E_{MEY}x_{MEY}.
\tag{2.14}
$$

Therefore, the optimal control problem is to maximize the present value (or discounted value) of the net revenue, and can be expressed as:

$$\max_E J(E) = \int_0^\infty e^{-\delta t} \left(pq(1+\theta)x - c_1 - \frac{c_2}{2}E \right) E \, dt$$

$$\text{subject to} \quad \frac{dx}{dt} = rx\left(1 - \frac{x}{K}\right) - q(1+\theta)Ex \tag{2.15}$$

$$x(0) = x_0$$

$$0 \le E \le E_{max}.$$

For this study, the biological parameter values employed are $r = 1.42/\text{year}$, $q = 1.8 \times 10^{-6}/\text{trip/year}$ and $K = 1 \times 10^6$ tonnes. The economic values are given by $p = \$600/\text{tonne}$ and $c_1 = \$195/\text{trip/year}$ [25]. In addition, the discount rate δ is assumed to be $0.15/\text{year}$. Note that the currency is denominated in United States dollars.

The linear and quadratic costs are depicted in Fig. 1. The perturbation in the linear costs is such that when the effort is at the MSY level, the quadratic costs are 25% greater than the linear costs. Therefore, c_2 is computed as

$$c_2 = 2\left(\frac{0.25c_1}{E_{MSY}}\right)$$

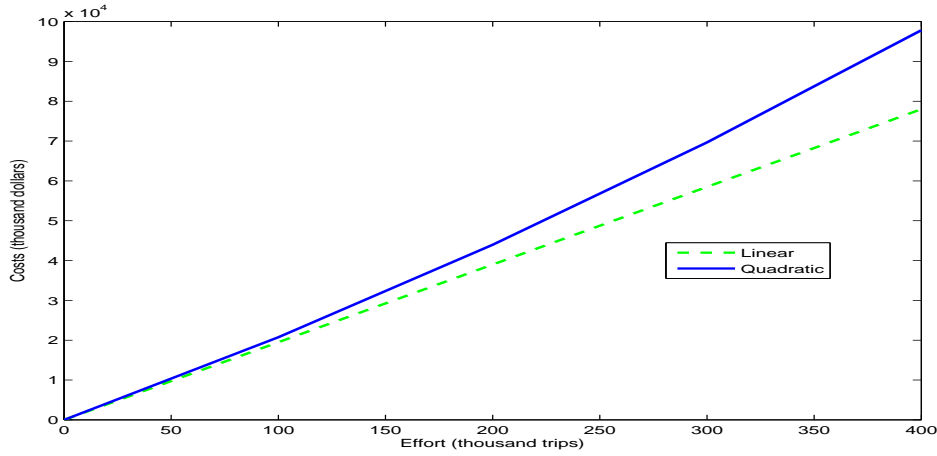$$= \$2.47 \times 10^{-4}/\text{trip}^2/\text{year}.$$

Fig. 1. Linear and quadratic costs

3   Bifurcation Analysis

When the parameter of a dynamical system is varied, it usually leads to a change in the number of equilibrium points or the stability properties of the system. This phenomenon is known as a bifurcation. The solution trajectories for the various scenarios depicting the effects of the variation in catchability on the fish stock are presented.
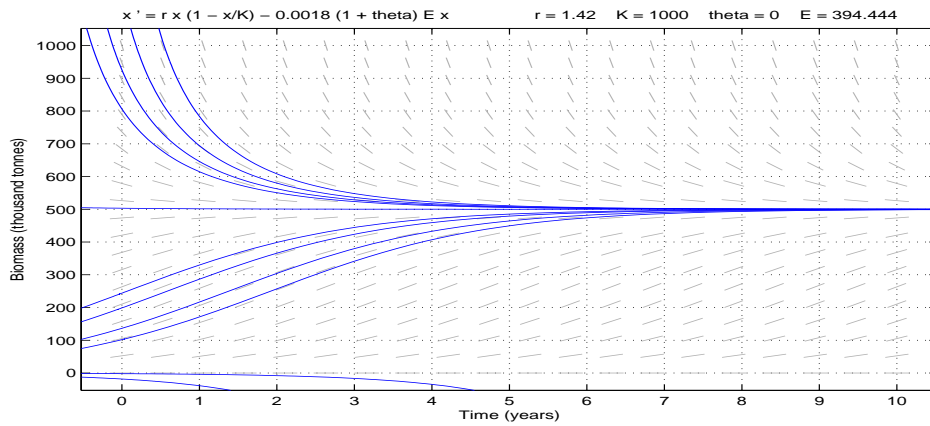
Fig. 2. Some solution trajectories when $E = 394,444$ and $\theta = 0$

The case where $E = E_{MSY} = 394,444$ trips with no variation in catchability, $\theta = 0$, is shown in Fig. 2. The system is structurally stable because there are two hyperbolic equilibrium points: 0 and $x_{eqm} = x_{MSY} = 500,000$ tonnes. For biomass levels $x_0 > x_{MSY}$ and $0 < x_0 < x_{MSY}$, the population asymptotically approaches x_{MSY}. Therefore, the zero stock size is unstable while x_{MSY} is stable. Furthermore, an effort rate of E_{MSY} leads to a stock size that is exactly half the carrying capacity.
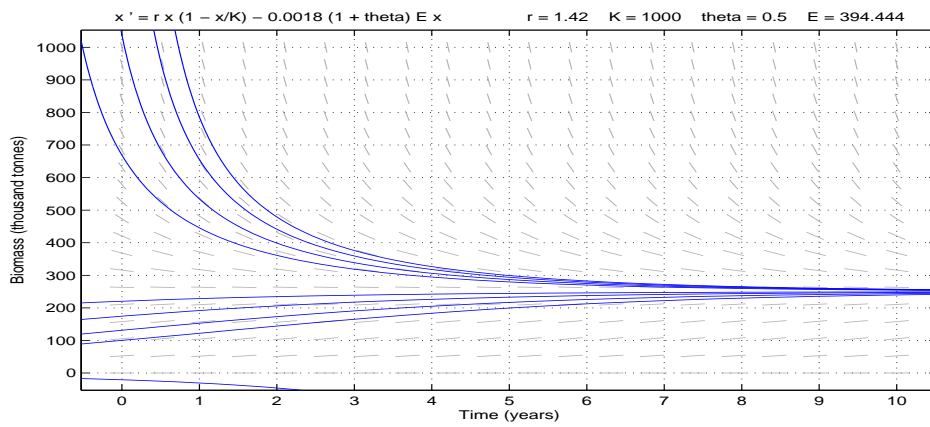

Fig. 3. Some solution trajectories when $E = 394,444$ and $\theta = 0.5$

Solution trajectories depicting a variation in catchability, $\theta = 0.5$, are presented in Fig. 3. There are two hyperbolic equilibrium points: 0, which is unstable and $x^\theta = 250,000$ tonnes, which is stable. For $x_0 > x^\theta$ and $0 < x_0 < x^\theta$, the population asymptotically approaches x^θ. This implies that an increase in catchability of 50% is equivalent to fishing at an effort rate E^θ that is one and a half times the effort rate at E_{MSY}. This induces a long-term decline in fish stocks to a level that is 50% of x_{MSY}. Note that, even though the effort rate $E = E^\theta = 394,444$ trips is greater than $E^\theta_{MSY} = 262,963$ trips, this model is structurally stable as the effort rate is still less than both the bifurcation points of the modified and standard models, $525,926$ trips and $788,889$ trips, respectively.
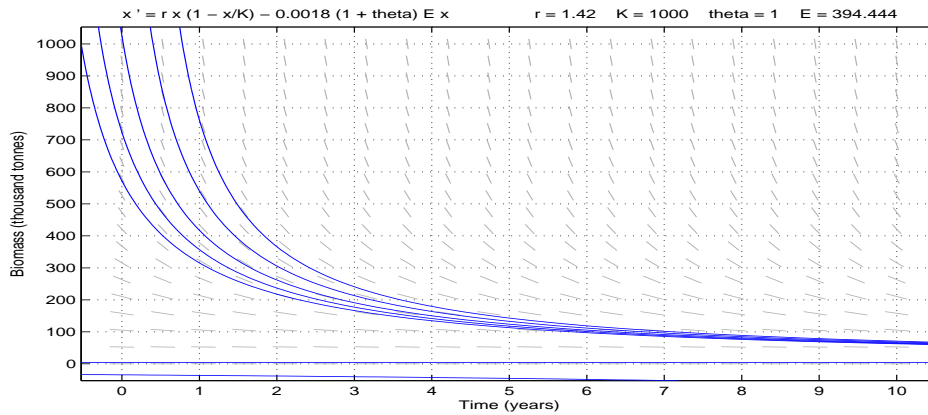
Fig. 4. Some solution trajectories when $E = 394,444$ **and** $\theta = 1$

Fig. 4. portrays an extreme scenario where the catchability is doubled ($\theta = 1$). This implies that an increase in catchability of 100% is equivalent to fishing at an effort rate that is twice the effort rate at E_{MSY} (see Equation (2.7)). Fishing at two times of E_{MSY} corresponds to the bifurcation point of the standard model. Thus, for any $x_0 > 0$ the population approaches the nonhyperbolic equilibrium population, 0. Therefore at the bifurcation point ($2 \times E_{MSY}$), the single equilibrium biomass level 0 is semi-stable (making the system structurally unstable). Hence, for any initial biomass level, the long-term population of fish stock is towards extinction. It is instructive to note that when $\theta = 1$, $E^{\theta} = 394,444$ trips exactly equal the bifurcation point of the modified model. However, to ensure sustainability of the resource ($x_{MSY} = x_{MSY}^{\theta} = 500,000$ tonnes) in the modified model, E_{MSY}^{θ} must be set $197,222$ trips (assuming zero costs and zero discounting).

4 Analysis of Optimal Control Problem

The sufficiency conditions are investigated and discussed in this section. In particular, the existence of an optimal control is determined. Also, the characterization of the optimal control and the existence and uniqueness of the optimality system are investigated.

4.1 Existence of optimal control

The stated goal is to maximize the present-value of the net revenue. Therefore, an optimal control E^* is sought that maximizes the objective functional over the Lebesgue measurable control set

$$U = \{E \,|\, 0 \le E(t) \le E_{max}, \ t \in [0, \infty)\}.$$

In the course of solving an optimal control problem, there is the need to investigate and verify necessary and sufficient conditions that ensure optimality of the problem. A sufficiency condition for the existence of an optimal control to Problem (2.15) is given in Theorem 4.1 [26, 27].

Theorem 4.1. *Given the control problem* (2.15)*, there exists an optimal control E^* that maximizes the objective functional $J(E)$ over the control set U if the following conditions are satisfied:*

(i) The class of all initial conditions with a control E in the admissible control set together with the state system is nonempty.

(ii) The control set U is closed and convex.

(iii) The right hand side of the state system is bounded above by a linear function involving the state and control variables.

(iv) The integrand of the objective functional is concave on U.

(v) There exist constants w_1, $w_2 > 0$ and $\eta > 1$ such that the integrand $f(t, x, E)$ of the objective functional satisfies
$$f(t, x, E) \leq w_1 - w_2 |E|^{\eta}.$$

Proof. To prove the theorem, the given conditions are established as follows:

Regarding the first condition, the Picard-Lindelof existence theorem [28] guarantees the existence and uniqueness of a solution to a state equation with bounded coefficients.

By definition, the control set U is closed and convex. This verifies condition 2. For verification of condition 3, the comparison theory of differential equations is applied to determine the boundedness of the solution to the state equation. Since
$$x' = rx\left(1 - \frac{x}{K}\right) - q(1 + \theta)Ex \leq rx\left(1 - \frac{x}{K}\right)$$

for $0 \leq t < \infty$ and $x_0 > 0$, then
$$x' \leq rx\left(1 - \frac{x}{K}\right) = rx - \frac{rx^2}{K}.$$

For $x' \geq 0$,
$$0 \leq rx - \frac{rx^2}{K},$$

and
$$0 \leq \frac{rx^2}{K} \leq rx.$$

Thus,
$$0 \leq x(t) \leq K.$$

Additionally, the right hand side of the state equation can be expressed as
$$S(t, x, E) = rx\left(1 - \frac{x}{K}\right) - q(1 + \theta)Ex \leq rx \leq rK.$$

Hence the bound on the right hand side can be written as
$$S(t, x, E) \leq rK.$$

To prove that the integrand of the objective functional is concave on U, let $f(t, x, E) = e^{-\delta t}L(t, x, E) = e^{-\delta t}pq(1 + \theta)xE - c_1 E - \frac{c_2}{2}E^2$. Then $f(t, x, E) \leq L(t, x, E)$, since $e^{-\delta t} > 0$ for $t \geq 0$.

Using the convex property of E, for $0 \leq m \leq 1$ and E_1, $E_2 \in U$, it implies that
$$mE_1^2 + (1 - m)E_2^2 \geq [mE_1 + (1 - m)E_2]^2.$$

Therefore, the objective is to show that, for $0 \leq m \leq 1$,
$$mf(t, x, E_1) + (1 - m)f(t, x, E_2) \leq f(t, x, mE_1 + (1 - m)E_2),$$

or
$$mL(t, x, E_1) + (1 - m)L(t, x, E_2) \leq L(t, x, mE_1 + (1 - m)E.$$

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This proof starts by observing that the difference of $mL(t, x, E_1) + (1-m)L(t, x, E_2)$ and $L(t, x, mE_1 + (1-m)E_2)$ is given by

$$mL(t, x, E_1) + (1 - m)L(t, x, E_2) - L(t, x, mE_1 + (1 - m)E_2)$$
$$= mpq(1 + \theta)xE_1 - mc_1E_1 - m\frac{c_2}{2}E_1^2 + (1 - m)pq(1 + \theta)xE_2 - (1 - m)c_1E_2 - (1 - m)\frac{c_2}{2}E_2^2$$
$$- pq(1 + \theta)x[mE_1 + (1 - m)E_2] + c_1[mE_1 + (1 - m)E_2] + \frac{c_2}{2}[mE_1 + (1 - m)E_2]^2.$$

Simplifying the right-hand-side gives

$$-\frac{c_2}{2}\{mE_1^2 + (1 - m)E_2^2 - [mE_1 + (1 - m)E_2]^2\} \le 0,$$

since from the convexity of E,

$$mE_1^2 + (1 - m)E_2^2 - [mE_1 + (1 - m)E_2]^2 \ge 0.$$

Hence

$$mL(t, x, E_1) + (1 - m)L(t, x, E_2) \le L(t, x, mE_1 + (1 - m)E_2).$$

This verifies condition 4.

The final verification is condition 5. Since x and E are bounded, there exists a $B > 0$ such that $x \le B$ and $E \le B$ on $[0, \infty)$, where $B = \max(K, E_{max})$. Therefore,

$$pq(1 + \theta)xE - c_1E - \frac{c_2}{2}E^2 \quad \le \quad pq(1 + \theta)B^2 - \frac{c_2}{2}E^2$$
$$\le \quad w_1 - w_2E^2,$$

where

$$w_1 = pq(1 + \theta)B^2, \quad w_2 = \frac{c_2}{2} \quad \text{and} \quad \eta = 2.$$

$\square$

To show that the objective functional is convergent as $t \to \infty$, let $w_1 - w_2E^2 = G$. Then

$$\int_0^\infty e^{-\delta t} \left(pq(1 + \theta)x - c_1 - \frac{c_2}{2}E\right) E \, dt \le \int_0^\infty e^{-\delta t}G \, dt = \frac{G}{\delta}.$$

4.2 Characterization of optimal control

In this section, the optimal control is characterized – obtaining an explicit formulation for the optimal control level – as well as determining the optimality system. Having established the existence of an optimal control to Problem (2.15), the necessary conditions for the control are derived using Pontryagin's maximum principle [29].

Theorem 4.2. *Given an optimal control E^* and a solution to the corresponding state equation, there exists an adjoint variable λ satisfying*

$$\lambda' = \left(\delta - r + \frac{2rx}{K}\right)\lambda - (p - \lambda)q(1 + \theta)E, \tag{4.1}$$

and the transversality condition

$$\lim_{t \to \infty} \lambda(t) = 0.$$

Furthermore, E^* can be represented as

$$E^* = \min\left(E_{max}, \left(\frac{(p - \lambda)q(1 + \theta)x - c_1}{c_2}\right)^+\right),$$

where the notation [27, 30] is given by

$$n^+ = \begin{cases} n & \text{if} \quad n > 0, \\ 0 & \text{if} \quad n \leq 0. \end{cases}$$

Proof. The current value Hamiltonian for the optimal control problem (2.15) is

$$H = \left(pq(1+\theta)x - c_1 - \frac{c_2}{2}E \right) E + \lambda \left[rx \left(1 - \frac{x}{K} \right) - q(1+\theta)Ex \right]. \tag{4.2}$$

Therefore, Equation (4.1) is obtained from the following adjoint equation:

$$\lambda' = \delta\lambda - \frac{\partial H}{\partial x}.$$

The optimality condition is given by

$$\frac{\partial H}{\partial E} = pq(1+\theta)x - c_1 - c_2 E - \lambda q(1+\theta)x = 0.$$

Thus,

$$E^* = \frac{(p-\lambda)q(1+\theta)x - c_1}{c_2}. \tag{4.3}$$

The characterization of the optimal control is

$$\begin{cases} E^* = 0 & \text{if} \quad \frac{\partial H}{\partial E} < 0, \\ 0 \leq E^* \leq E_{max} & \text{if} \quad \frac{\partial H}{\partial E} = 0, \\ E^* = E_{max} & \text{if} \quad \frac{\partial H}{\partial E} > 0. \end{cases}$$

Employing standard arguments regarding the bounds on the control, the following ensures:

$$E^* = 0 \quad \text{if} \quad \frac{\partial H}{\partial E} < 0.$$

This implies

$$pq(1+\theta)x - c_1 - c_2 E - \lambda q(1+\theta)x < 0,$$

and so

$$\frac{(p-\lambda)q(1+\theta)x - c_1}{c_2} < E^* = 0.$$

Thus,

$$\frac{(p-\lambda)q(1+\theta)x - c_1}{c_2} < 0.$$

Similarly,

$$E^* = E_{max} \quad \text{if} \quad \frac{\partial H}{\partial E} > 0.$$

This shows that

$$pq(1+\theta)x - c_1 - c_2 E - \lambda q(1+\theta)x > 0,$$

implying

$$\frac{(p-\lambda)q(1+\theta)x - c_1}{c_2} > E^* = E_{max}.$$

Thus,

$$\frac{(p-\lambda)q(1+\theta)x - c_1}{c_2} > E_{max}.$$

Therefore,

$$E^* = \begin{cases} 0 & \text{if} \quad \frac{(p-\lambda)q(1+\theta)x - c_1}{c_2} < 0, \\ \frac{(p-\lambda)q(1+\theta)x - c_1}{c_2} & \text{if} \quad 0 \leq \frac{(p-\lambda)q(1+\theta)x - c_1}{c_2} \leq E_{max}, \\ E_{max} & \text{if} \quad \frac{(p-\lambda)q(1+\theta)x - c_1}{c_2} > E_{max}. \end{cases}$$

Hence, the optimal fishing effort can be written as

$$
E^* = \begin{cases}
0 & \text{if} \quad \lambda > p - \dfrac{c_1}{q(1+\theta)x}, \\[3mm]
\dfrac{(p-\lambda)q(1+\theta)x - c_1}{c_2} & \text{if} \quad p - \dfrac{(c_1 + c_2 E_{max})}{q(1+\theta)x} \le \lambda \le p - \dfrac{c_1}{q(1+\theta)x}, \\[3mm]
E_{max} & \text{if} \quad \lambda < p - \dfrac{(c_1 + c_2 E_{max})}{q(1+\theta)x}.
\end{cases}
$$

Therefore, the optimal control consists of both the boundary solution (where constraints are binding) and the interior solution. The former implies that the resource should be harvested provided that the net revenue per unit harvest (or marginal revenue of harvest) due to the application of maximum effort exceeds the current value shadow price of the resource (or marginal revenue of stock) [31].

In compact form,

$$
E^* = \min\left(E_{max}, \left(\dfrac{(p-\lambda)q(1+\theta)x - c_1}{c_2} \right)^+ \right).
$$

The optimality system consists of the characterized optimal control, state and adjoint equations together with the initial and transversality conditions [32].

Therefore,

$$
x' = rx\left(1 - \frac{x}{K}\right) - q(1+\theta)\min\left(E_{max}, \left(\frac{(p-\lambda)q(1+\theta)x - c_1}{c_2} \right)^+ \right)x,
$$

$$
\lambda' = \left(\delta - r + \frac{2rx}{K}\right)\lambda - (p-\lambda)q(1+\theta)\min\left(E_{max}, \left(\frac{(p-\lambda)q(1+\theta)x - c_1}{c_2} \right)^+ \right),
$$

$$
\text{with} \quad x(0) = x_0 \quad \text{and} \quad \lim_{t\to\infty} \lambda(t) = 0.
$$

$\square$

The uniqueness of the optimal control is investigated, since Theorem 4.1 establishes the existence of the control. Given the a priori boundedness of the state and adjoint equations together with the state equation being continuously differentiable, the mean value theorem ensures that the Lipschitz condition is satisfied by the state equation with respect to the state variable. This guarantees the uniqueness of the optimality system for small time intervals as result of the opposite time orientations of the state and adjoint equations. Furthermore, the uniqueness of the solutions of the optimality system guarantees uniqueness of the optimal control [32, 33, 34].

5 Simulations

Simulations are carried out using the Forward-Backward Sweep method outlined in Lenhart and Workman [32]. As the name of the method indicates, the state equation is solved forward in time while the adjoint equation is solved backward in time in order to achieve convergence.

In the standard Gordon-Schaefer model (where $\theta = 0$ and $c_2 = 0$) the optimum sustainable yield (OSY) is found to be $351,328$ trips per annum [19]. Therefore to ensure sustainability of the resource, the annual effort rate of the quadratic model should be less than $351,328$ trips (since the higher fishing costs in the quadratic model have the tendency to reduce effort rate). To ensure the sustainability of the quadratic model, the sustainable yield (SY) must be pegged at $315,000$ trips per annum.

The model is subjected to numerical simulations with the maximum fishing effort initially set at the SY level, E_{SY} and the results illustrated graphically. Also considered is the case of E_{MEY}^{θ} with no variation in catchability. That is, let E_{MEY}^{θ} with $\theta = 0$ and $c_2 \neq 0$ be represented by $E_{MEY}^0 = 296,380$ trips (see Equation (2.12)). Firstly, simulations are carried out with a fixed initial biomass level (and no variation in catchability) while varying the rate of fishing effort. Secondly, the model is simulated for a fixed rate of fishing effort while varying the catchability coefficient to take into account the illegal and unapproved fishing methods practiced by the fishermen.

In the simulations, a time horizon of 20 years was employed in scenarios where the fishery was found to be sustainable; that is, achieved equilibrium. The transversality condition ensures that the shadow price at the terminal time T is zero, since any resource not depleted at the end of the planning horizon must have zero value [35].

5.1 Long-run dynamics of model

The long-run scenario, as shown in Fig. 5, depicts fishing at a maximum effort rate of $315,000$ trips (SY effort rate) and an initial biomass level of $550,000$ tonnes. From an initial value of $\$346.35$, the shadow price steadily decreases and after a few years sharply declines to zero. However, the net revenue of $\$346.35$ is almost constant for the entire horizon. The fact that the shadow price is lower than the net revenue for the majority of the horizon indicates that the marginal revenue of harvest exceeds the marginal revenue of stock. It is therefore in the fishermen's best interest to apply the maximum available effort in harvesting.
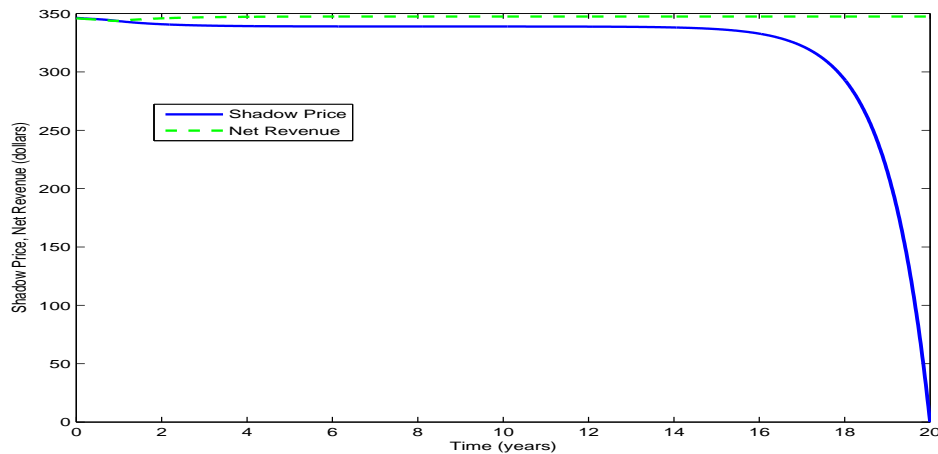

Fig. 5. Shadow price and net revenue for $x_0 = 550,000$, $\theta = 0$ and $E_{max} = 315,000$

In Fig. 6, it is observed that when the maximum effort rate E_{max} is set at the MEY and SY levels, the optimal effort rates settle down at their respective equilibrium levels. Starting at about $245,827$ trips, the effort rate for E_{MEY}^0 increases rapidly and stabilizes at a final value of around $296,310$ trips. Meanwhile, the effort rate for E_{SY} starts much lower at $227,634$ trips and converges to $314,923$ trips. Similarly, the biomass levels for MEY and SY increase and converge to the equilibrium values of $624,491$ tonnes and $600,899$ tonnes respectively. The total net revenues corresponding to the effort levels at MEY and SY are computed as $\$806,620,000$ and $\$809,690,000$, respectively.
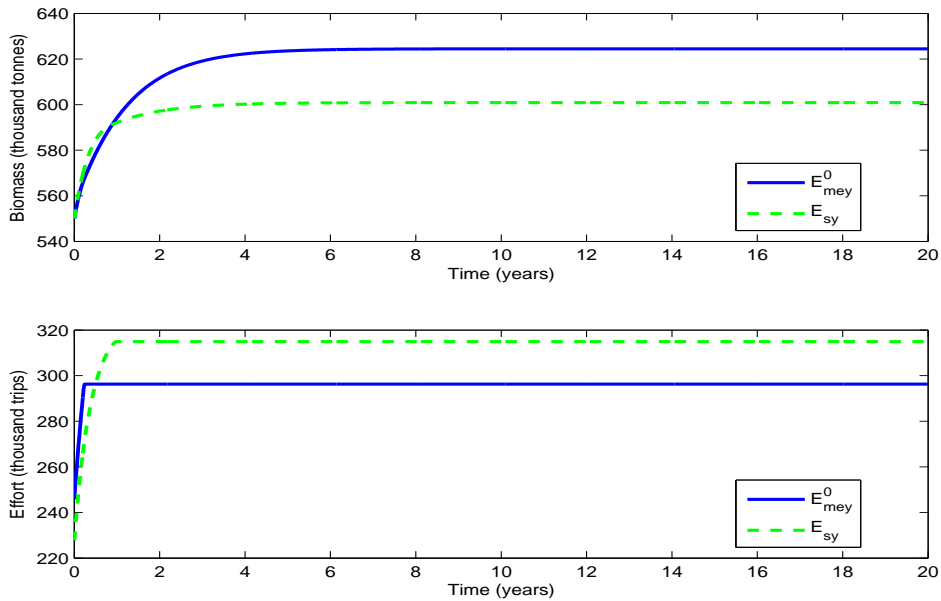
Fig. 6. Effort strategies and biomass levels for $x_0 = 550,000$, $\theta = 0$ and $E_{max} = 296,380$ versus $E_{max} = 315,000$

Fig. 7 shows that the initial shadow price, \$220.92, is significantly lower than the net revenue, \$455.06. Furthermore, at the final horizon, the net revenue is \$440.90 while the shadow price tends to zero. This shows that it is optimal to exert the maximum effort for the entire horizon, as the revenue far exceeds the shadow price.
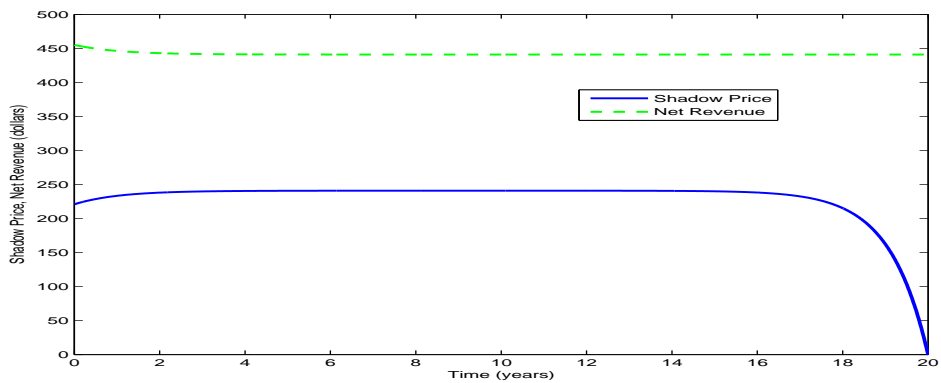

Fig. 7. Shadow price and net revenue for $x_0 = 750,000$, $\theta = 0.25$ and $E_{max} = 200,000$

In Fig. 8, the optimal effort rates follow the same path of around $200,000$ trips regardless of the catctability level of the fishery. However, the fish biomass levels decrease for both catchability levels $\theta = 0$ and $\theta = 0.25$ to their respective equilibrium values $746,603$ tonnes and $683,253$ tonnes. This shows that an increase in catchability while maintaining the same fishing effort has an adverse effect on the resource biomass. The total net revenues for $\theta = 0$ and $\theta = 0.25$ are \$743,540,000 and \$905,340,000, respectively. In other words, a 25% increment in the catchability results in a revenue increase of about 22% and a decrease in final biomass level of 8%.
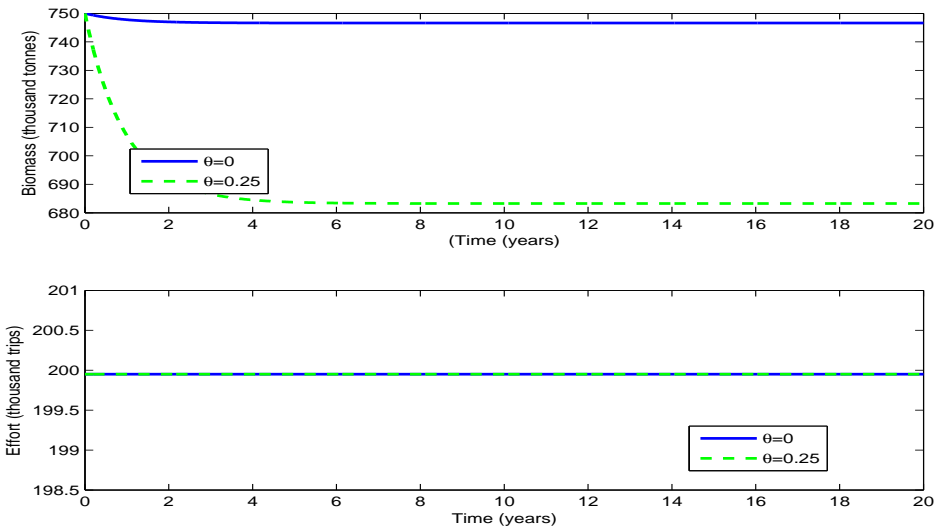
Fig. 8. Effort strategies and biomass levels for $x_0 = 750,000$**,** $E_{max} = 200,000$ **and** $\theta = 0$
versus $\theta = 0.25$

Fig. 9. shows that the optimal effort rates follow the same trajectory of around $200,000$ trips for both catchability levels throughout the horizon. On the other hand, the fish biomass levels decrease for the catchability levels $\theta = 0.5$ and $\theta = 0.75$ to the equilibrium values $619,904$ tonnes and $556,555$ tonnes, respectively. The total net revenues corresponding to $\theta = 0.5$ and $\theta = 0.75$ are respectively $\$1,031,000,000$ and $\$1,121,500,000$. Thus, a 50% increment in the catchability results in a revenue increase of 9%, and a decrease in final biomass level of 10%.
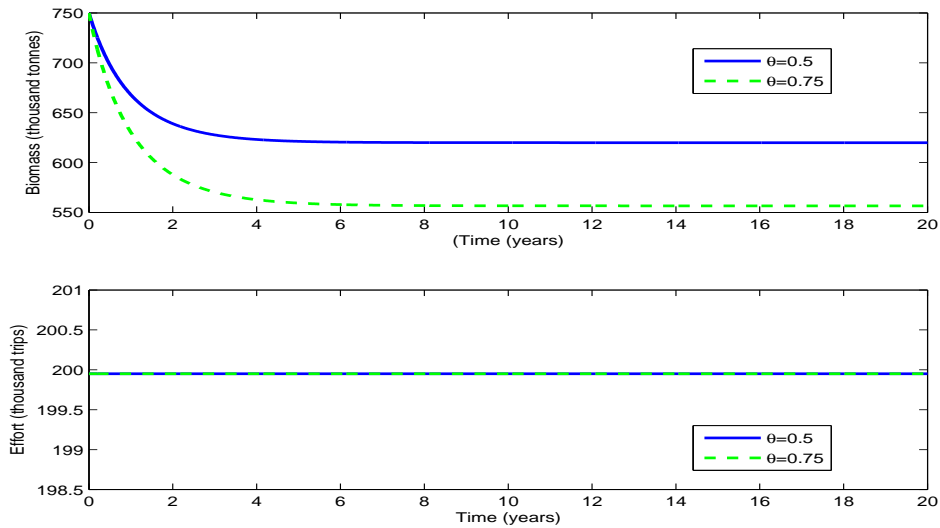

Fig. 9. Effort strategies and biomass levels for $x_0 = 750,000$**,** $E_{max} = 200,000$ **and** $\theta = 0.5$
versus $\theta = 0.75$

Fig. 10. shows that the initial shadow price, \$346.14, is significantly lower than the net revenue, \$509.41 making it worthwhile to harvest at the maximum effort rate. After some years, the shadow price and the net revenue attain the same value of \$469.25. Subsequently, the shadow price plummets to zero while the net revenue ends at \$462.60. Thus, the optimal control alternates between the interior and boundary controls. This shows that it is not optimal to exert the maximum effort at the middle portion of the horizon.
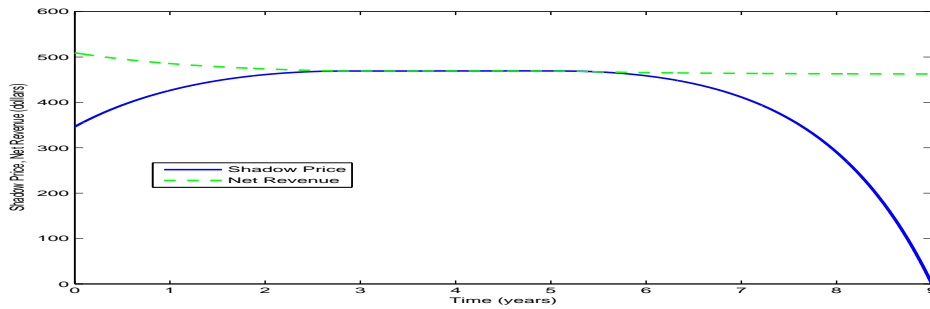

Fig. 10. Shadow price and net revenue for $x_0 = 750,000$, $E_{max} = 200,000$, $\theta = 1$ **and** $T = 9$

In Fig. 11, the optimal effort rates follow the same trajectory of almost $200,000$ trips for both catchability levels throughout the nine-year horizon, except for a brief period where it is almost convex and attaining a minimum value of $190,748$ trips for $\theta = 1$. The biomass decreases for the catchability levels $\theta = 0.75$ and $\theta = 1$ to $556,672$ tonnes and $494,515$ tonnes, respectively. It is noteworthy that when $\theta = 1$, the iterates failed to converge beyond a time horizon of nine years. The total net revenues for $\theta = 0.75$ and $\theta = 1$ are \$889,200,000 and \$941,440,000, respectively. Therefore, a 33% increment in the catchability results in a revenue increase of 6%, and a decrease in final biomass level of 11% (with no equilibrium or sustainable level achieved for $\theta = 1$).
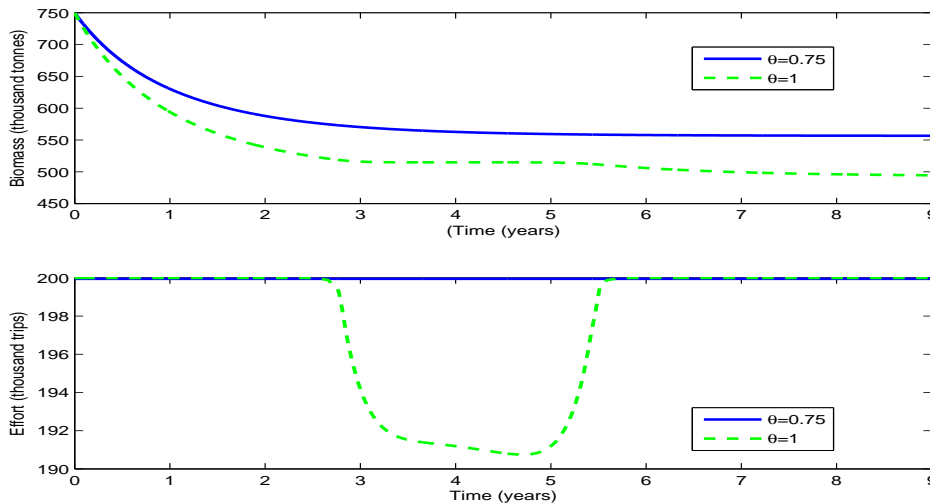

Fig. 11. Effort strategies and biomass levels for $x_0 = 750,000$, $E_{max} = 200,000$ **and** $\theta = 0.75$ **versus** $\theta = 1$

5.2 Sensitivity analysis on discount rate

The plots in Fig. 12. portray the net revenue against the modification on the catchability coefficient θ for a time horizon of twenty years. The figure depicts a scenario where the discount rate δ varies from 0 to 15% per year. When δ is 15% per year, the net revenue curve is concave, starting from a value of \$743,540,000 at $\theta = 0$ and increasing to a maximum value of \$1,106,200,000 at $\theta = 0.7$.

Similarly, when the discount rate δ is 0 per year, the net revenue curve is concave, starting from \$2,346,300,000 and increasing to a maximum of \$3,371,700,000 at $\theta = 0.7$. The curve for $\delta = 0$ is always above the curve for $\delta = 0.15$ for the given values of θ. In addition, the maximum revenue when $\delta = 0$ is 67% greater than when $\delta = 0.15$.
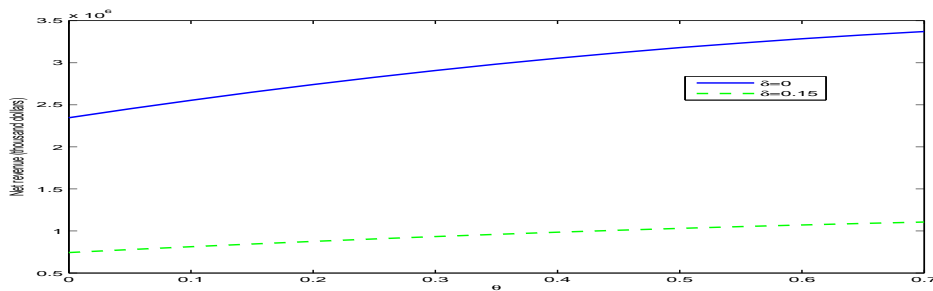

Fig. 12. Plot of net revenue against θ for $T = 20$ and $\delta = 0$ versus $\delta = 0.15$

6 Concluding Remarks

This work has investigated the fishing effort strategies for the sardinella fishery under the modified Gordon-Schaefer model in order to determine the optimal strategy. Dynamics of the fish biomass were modeled using a modified Schaefer equation, and bifurcation analysis performed on this model with a variation in the catchability coefficient. Furthermore, the objective functional of the canonical Gordon-Schaefer model was subjected to a modification. Instead of the linear costs in the model, a more realistic cost option – quadratic costs – was considered. The reference points under this modified model, namely the MSY, MEY and OAY, were determined. It was realized that when the proportion of variation in catchability is zero and the coefficient of the quadratic cost term is also zero, the reference points for the modified model reduce to the standard Gordon-Schaefer model reference points. The existence of an optimal control was proven as well as the control characterized using Pontryagin's maximum principle. Uniqueness of the optimality system is guaranteed due to the Lipschitz property of the model.

Numerical simulations were carried out on the modified model, with the quadratic costs seen as perturbations on the usual linear costs. Sensitivity analysis was performed on the catchability coefficient to simulate the effects of IUU fishing (especially the use of under-sized mesh gears) on fish stocks. This study has shed light on the contribution of IUU fishing to the near-collapse of the sardinella fishery in Ghana. The simulation results show that in the long run, IUU fishing has a disastrous effect on fish biomass with minimal increase in total net revenue. At higher levels of increased catchability as a result of the illegal practices, the consequences on the fish stock size are near catastrophic levels. That is, the fish stocks are driven to less than half of the carrying capacity of the ecosystem in finite time without any huge benefits in terms of additional revenue to the fishermen. Therefore, to ensure the long-term sustainability of the resource, fishery managers should strictly enforce all the regulations pertaining to the use of under-sized mesh gears and other IUU fishing practices.

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Competing Interests

Author has declared that no competing interests exist.

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