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A Modeling Study of Microscale Convection on the Heat Transfer Mode of a Fluid

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Abstract

This paper deals with microscale convection on the heat transfer mode associated with the bulk movement of a fluid. It occurs when a fluid is in contact with a solid surface, and when there is a temperature difference between the surface and the fluid. Whether the motion of the fluid particles is due to density differences imposed by a temperature distribution in the fluid and the resultant buoyancy device, as in the case of face and natural convection, or due to an external device such as a pump that forces the fluid pass the surface, the energy transport phenomenon in convection in always closely related to the fluid motion. Thus, the analysis of convective heat transfer involves a careful analysis of the fluid flow, its behaviour, and characteristics.

Keywords: Convective heat transfer; convective heat transfer in microtubes and channels; electric double layer; nonconventional analysis method;

Nomenclature

E_+ complex vector; E: energy of a quantized energy packet; f: particle distribution function; f_o Fermi Dirac distribution function; H: magnetic field; H: height; h: Planck constant; q: wave vector; Nu: Nusselt number; Re: Reynolds number; S: fin spacing.

Greek Symbols

ν	frequency
τ	relaxation time
μ	magnetic permeability
σ	accommodation coefficient
σ	characteristic distance

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ρ electrical resistivity
ω angular frequency

Subscripts
sc: scattering

1 Introduction

Convective heat transfer in fluids through channels of very small dimensions has been proposed as an effective method of heat dissipation from substrates used in electronic components as early as the 1980s. While high heat transfer rates and larger pressure drops were measured, measurements using the available probes and instrumentation techniques indicated that conventional correlations for convective heat transfer were not adequate to predict the measure frictional and heat transfer data. A number of theoretical and experimental approaches have been applied to analyse the convective heat transfer problem, in order to design guidelines for convective cooling systems. On the other hand, in conventional microscale convective systems, the theoretical approaches invariably consists the mathematical formulation of the problem using conservation principles for the flow and heat transfer processes and solution of the governing differential equations, which are mathematical statements of these principles. The set of differential conservation equations for mass, momentum, and energy applied for a control volume, coupled with the appropriate boundary conditions, constitute the most common theoretical descriptions of convective heat transfer. Integral forms of the equations can be used to obtain approximate solutions for the for the flow and heat transfer fields. Solution of the equations, which govern and describe the fluid flow and heat transfer phenomena using analytical or numerical methods, yields the distributions of the field variables in a fluid continuum, such as the velocity components in the chosen coordinate directions, and temperature distributions and density distributions in case where these apply. The velocity distribution in the flow field is then used to compute the frictional shear stresses, and the temperature distributions, which in turn can be used to compute the surface heat transfer rates, through a calculation of field gradients and application of differential form of Fourier's law. The friction coefficients and Nusselt numbers, which are useful nondimensional parameters, for describing the frictional and heat transfer effects, can also be computed and correlated with the flow parameters and fluid properties using such analytical techniques. Furthermore, some of the aspects of convective heat transfer analysis are investigated and are applicable when small channels are examined. However, the major findings related to convective heat transfer, with single-phase flow, phase change, and two-phase flow, and gas flow in microchannels are reviewed in the light of the published literature.

2 Microscale Heat Transfer: A Recent Avenue in Energy Transport

The influence of the physical size of the domain under consideration, or the extent of the medium, on heat transfer has been interest to scientists for 400 years. Whether it is heat conduction in solids, where the surface boundaries or interfaces can be significance; convective heat transfer in fluids, where the basic mechanism is conduction into a fluid layer adjacent to a surface; or radiative heat transfer, where the heat transport mechanism is electromagnetic in nature, size effects are important, particularly when dealing with small- length- scale systems. While the fundamental physics within the physical domain of microscale dimensions are typically the same, the small-length-scale effects often make conventional approaches to the analysis of heat transport at small-length-scales inappropriate or, in some cases, inaccurate. Though the fluid flow and heat transfer in channel with very small hydraulic mean diameters have been interest for quite some time, it has only been recently that the study of microscale thermal phenomena in engineering applications has gained relevance and importance. In addition, microscale conduction and radiative heat transfer have become an important area of study, due to the numerous applications involving the fabrication processes and performance monitoring of semiconductor devices. Focused studies have been reported over a broad range of microscale thermal transport phenomena, and the depth and breath of knowledge has increased dramatically. More recently, due to the significant potential of applications in micromachining, microelectronics, and microelectromechanical systems (MEMS), the field has expanded into the domain of nanoscale heat transfer, which calls for an altogether different approach and analysis methodology to clearly understand and apply the physical phenomena associated with thermal transport at these length scales.

3 Microchannel Flow and Convective Heat Transfer

The classification of channels into large, mini, and micro is quite arbitrary in nature. Over the past few years, because of a large number of research publications appearing in the literature on fluid flow and heat transfer in channels of varying size, general method of classification has evolved. This has resulted in a terminology or classification such that channels for which the largest dimensions of the polygonal cross section in below1 mm are classified as microchannels. Channels with cross section is below 1 mm are the microchannels. On the other hand, channels with cross sectional dimensions in the range of a few millimeters are termed as minichannels (1-5 mm) and those above considered as conventional of large channels. Furthermore, of the fundamental modes of heat transfer the one most studied in relation to microscale heat transfer is convective heat transfer in microchannels. The pioneering in the application of small channels for heat dissipation from silicon substrates was investigated (Tuckerman and Pease,1981) and this investigation focused on the design and testing of very compact water-cooled heat sinks integrated into silicon substrates. The channels used in these heat sinks integrated were as small as 50 µm wide and 300 µm deep, which are typical of microchannels in use today and are characterized by dimensions less than a millimeter. The first effort held the promise of using fluid flow through microchannles as an effective means of heat dissipation from silicon integrated circuits. We have discussed a very brief overview of the early studies here and studies on microchannel convection has resulted in findings that were mutually contradictory in both the values and the trends. Now, a comparision of these investigations, particularly those in the last decades of the twentieth century is presented in Fig. (1) and Fig. (2). It is observed from the figures both on fluid friction and on heat transfer, shows large variation, and also indicates significant deviations from the values predicted using well-accepted theoretical model and algorithms.

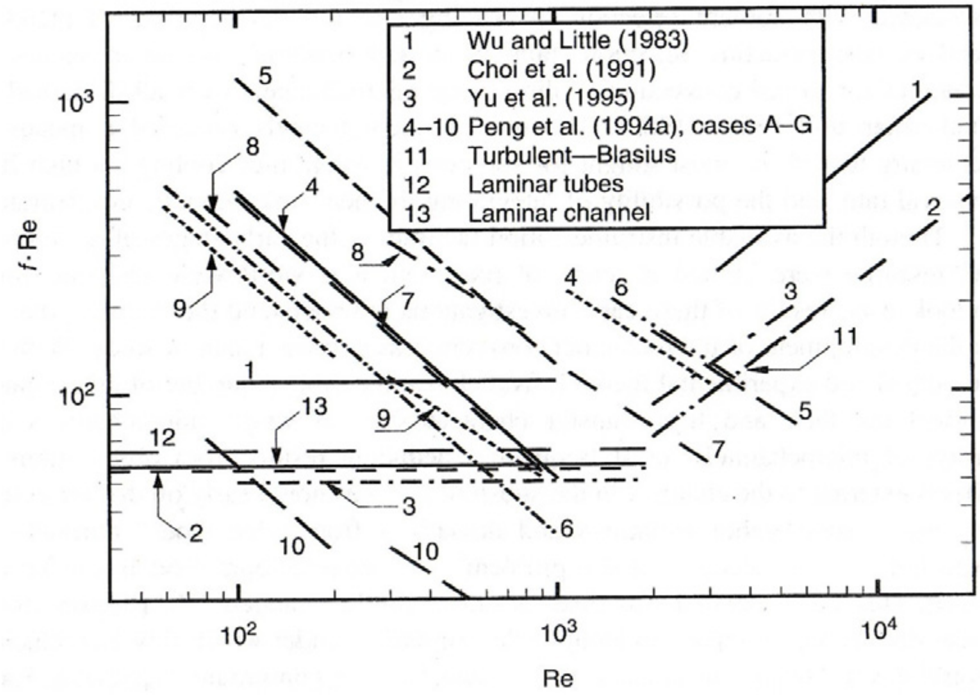

Fig. 1. Comparison of the fluid friction characteristics from various investigations on convective flow in microchannels

As the conventional predictions, the product of the friction factor and the Reynolds number (f × Re) should be constant for laminar flow (64 for circular and other constant values for other cross sections). For turbulent flow, f × Re should increase linearly while plotted on a log-log scale against the Reynolds number, for conventional turbulent flow as

$$f = 0.014\mathrm{Re}^{-0.182} \tag{1}$$

But, derivations from these expected results were reported for microchannels as is apparent in the findings presented in the Fig.(1). Now, no conclusive remarks can be made on the variations of $f \times \mathrm{Re}$ depicted in the plots, which as indicated, exhibit increasing and decreasing trends, both in the presumably laminar and the turbulent flow regimes. Again it is observed that the deviations and variations were also observed in the heat transfer characteristics, as in Fig. (2). On the other in the laminar flow regime it was observed that the dependence of the Nusselt number on the Reynolds number is generally stronger for the case of microchannels, compared to conventional channels as indicated by the by the steeper slopes of the plots. Further, the general trends reported here various investigations differed significantly, some of them lying below and some above the values predicted by conventional correlations, both in the laminar and turbulent flow regimes. But, another observation from these early experiments was that it appeared that the transitions from laminar flow was not characterized by the Reynolds number (based on the hydraulic diameter) criterion of 2000-2300, as would be expected from previous theory and empirical correlations. Again the critical Reynolds number characterized by the change in trends of the plots in Fig. (1), which determines the upper boundary of the laminar flow regime, was found to deviate from this theoretically predicted range of values, and variations depended on the hydraulic diameters of the channels used. Though it was not clear whether the deviations and differences were due to actual physical effects or limitations in fabrications and instrumentations, these above mentioned results paved the way for more serious thinking and closer observations of the problem of fluid flow and heat transfer in microchannels.

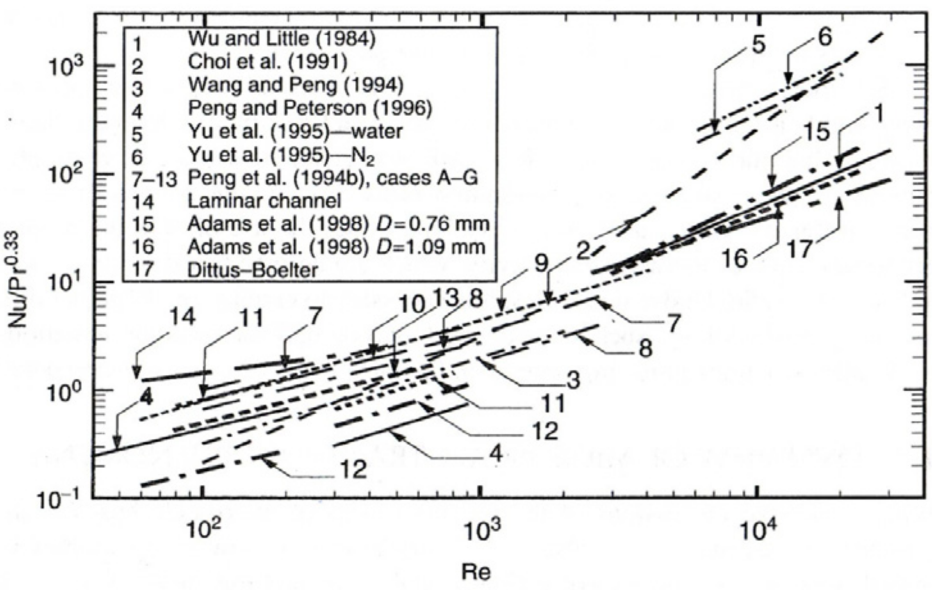

Fig. 2. Comparison of the heat transfer characteristics from various investigations on convective flow in microchannels

4 Phase Change and Two Phase Flow

A majority of the investigations of phase change ant two-phase flow in microchannels have been directed toward the cooling of electronic equipment. The objectives of our study were to determine and minimize the overall thermal resistance, as well as to obtain and correlate critical heat flux data. Friction models have been developed for two-phase liquid-vapour flows. Visualization studies on flow regimes have been presented by a number of investigators. Comparisons between the effectiveness of single-phase and two-phase cooling method have also been presented in the context of electronics cooling.

On the other hand, some of the early investigations (Bowers and Mudawar, 1994) were on circular mini-and microchannels, and were designed to obtain the critical heat flux values and optimize the channel thickness to diameter ratio as a compromise between the structural and heat transfer considerations. Pressure drops model were also developed for mini and microchannels and the theoretical predictions were compared with the experimental results. Furthermore, these investigations were intended to help determine and explain the heat transfer and pressure drop characteristics in relation to the flow acceleration, due to the phase change, and the channel erosion, due to how boiling. Studies directed at the exploration of the effects of parameters such as the flow velocity, subcooling, property variations, and channel configurations on two-phase flow behaviour were also undertaken. But, investigations indicated that some of the characteristics differed from those anticipated in conventional channels. In addition some of the early experiments (Peng et al., 1995) on nucleate flow boiling indicated that the liquid velocity and subcooling did not affect fully developed nucleate boiling, but greater subcooling increased the velocity and suppressed the initiation of flow boiling. Experiments of flow boiling of binary water/methanol mixtures in microchannels (Peng et al., 1996) indicated that the heat transfer coefficient at the onset the flow boiling and in the partial nucleate boiling region was greatly influenced by liquid concentration, microchannel and plate configuration, flow velocity, and subcooling, but these parameters has little effect in the fully nucleate boiling regime. Similar investigations of V-shaped microchannels (Peng et al., 1998) gave an impression that in contrast to conventional channels, no bubbles were observed in these microchannels during flow boiling, even with heat fluxes as high as 10^6 W/m^2. Again investigations using single and two phase flow experiments on refrigerants flow in microchannel heat exchangers (Cuta et al., 1996) inferred that a substantial improvement in thermal performance could be achieved in microscale heat exchangers without a large increase in pressure drop, though conclusive explanations were not offered for this observation. In addition recent experimentations obtained using flow visualization studies, however, have resulted in a dramatic modification in the understanding of the phase change and two-phase flow phenomena in these microscale channels. It is important to mention here that various new mathematical modeling techniques were also utilized and predictions very close to the experimental findings were obtained as in Fig. (3), particularly on the onset of nucleate boiling in small channels (Ghiaasiaan and Chedester, 2002).

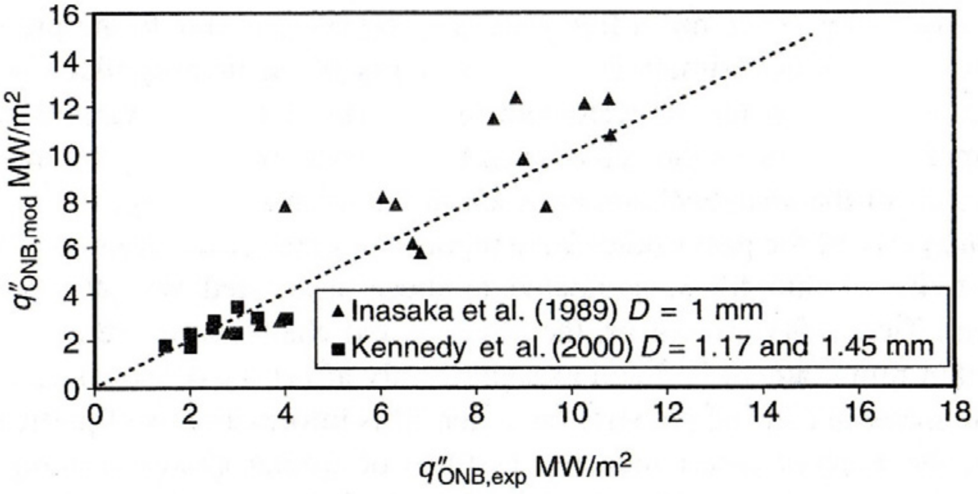

Fig. 3. Comparison of microtube ONB data with predictions from theoretical model

Furthermore, separate and distinct boiling, condensation and the associated two-phase flows have also been studied extensively more recently (Garimella et al 2005). Hybrid models combining experimental data and theoretical modeling have been developed to address the progression of the condensation process from the vapour phase to the liquid phase, including the overlap and transition between flow regimes.

5 Conduction and Radiation in the Microscale

In conduction and radiation size effects in electron transport phenomena, which has been observed when the characteristic dimensions (a thin metallic films and wires) become small, and the applications of thin films have been extensive in semiconductor technology, as microelectronics utilizing silicon-on-insulator (SOI) devices have more prevalent as these devices are being more widely utilized. The manufacture, as well as the operation of these devices, makes the thermal management problem one of the key challenges. Analysis of conduction of heat transfer in microstructure is essentially required for the prediction and design of the thermal effects in these types of systems. In addition, microscale conduction with pronounced size effects is encountered in structures where the characteristic length dimensions are of the same order as the scattering mean free path of the electrons. In this region, reductions in the transport coefficients, such as the thermal conductivity, are expected as a result of the shortening of the mean free path near the surface, due to the presence of the boundary. One of the primary goals of a majority of the investigations in microscale conduction has been the measurement, or estimation through various microscopic mathematical models, of the behaviour of the thermal conductivity when the physical dimensions of the analyzed structure are in the microscale range.

A majority of the past studies have reported a significant reduction in the thermal conductivity of thin films, compared to those associated with the bulk material (Flik and Tien, 1990). However, the physical and chemical composition of the thin film material is found to significantly affect the degree of deviation from the bulk material thermal property behaviour. In addition to experimental measurement, the theoretical analysis of microscale conduction is of critical importance in understanding the behaviour of these systems. Furthermore, a comprehensive analysis of microscale conduction systems should take into account the thermal behaviour associated with the film material used for the component or the structure. But (Kumar and Vradis, 1991) to use fundamental transport equations such as Boltzmann equation have indicated that the reduction in thermal and electrical conductivities in microscales was virtually identical in most of the practical cases for thin films. Fundamental approaches such as the use of the Boltzmann equation or molecular dynamics simulation were found to provide useful insight into the transport property behaviour of thin films and microscale physical structures. Theoretical maps defining the microscale and microscale regimes have proven useful for the thermal analysis of certain materials and take into account factors, such as material, purify and defect characteristics. Attempts have also been made to approach the problem of microscale conduction theoretically using modified (hyperbolic) heat conduction equations, in place of the conventional Fourier heat conduction analysis. Some representative results from these theoretical analyses (Ju and Goodson, 1999) are shown in Fig. (4).

New hypotheses, other than lattice vibrations, have been put forward to describe energy transport process, such as wave theory transport occurring in the conducting medium. Wave-type energy has been detected in metals such as gold films, while the heat transfer process takes place under lengths scale as low as 10^{-9} m (nanoscale) and timescales of the order of 10^{-12} s.

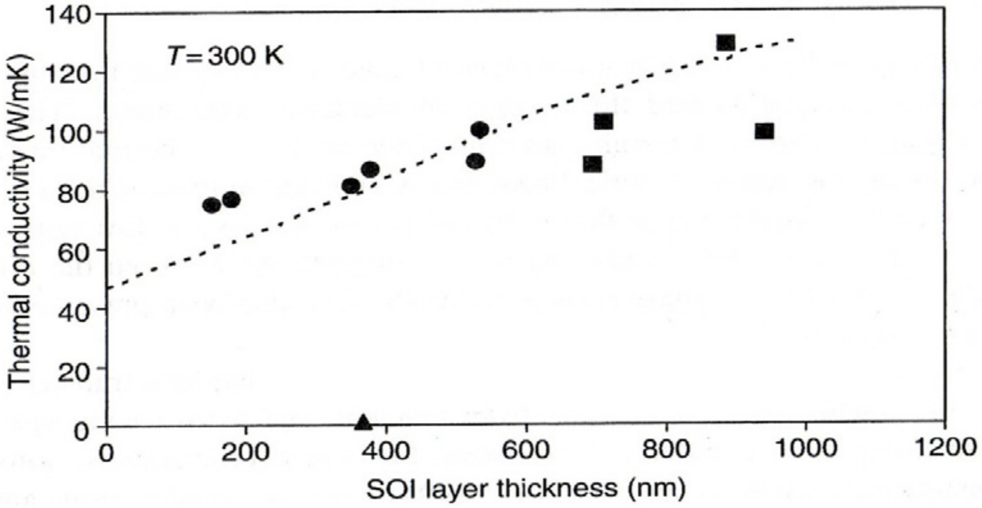

Fig. 4. Variation of the in-plane thermal conductivity of silicon layers with respect to the layer thickness at 300K, from various investigations

In addition thermal conductivities measurements in thin copper films, in a thickness range of 400-8000 Å and a temperature range of 100-500 K, and calculation of thermal conductivity of films using electrical analogies (Nath and Chopra, 1974), investigated the reduction of thermal conductivity with a reduction in the film thickness, while the temperature dependence agreed with variations in bulk copper. The effort taken (Flik et al., 1991) to study thermal transport in domains with various length scales to provide information to the designer as to whether conventional theory could be applied to a given microstructure is noteworthy. These investigations led to resign maps, which identify boundaries between microscale and macroscale heat transfer regimes, relating the smallest geometric dimension to the temperature. Additional maps have been developed for conduction, convection and radiation modes of heat transfer; a typical microscale regime map for conduction heat transfer is shown in Fig. (5) to demonstrate its application. These can be used to determine whether microscale heat transfer consideration is required to analyze and design microstructure pertaining to a given technology, by observing and positioning it appropriately on the map. Furthermore, an interesting finding is reported in the literature (Flik et al., 1991), which could serve as a guide in assessing whether conduction in thin films should be analyzed as a microscale conduction problem or not. This was because thermal conduction in a film is microscale, if the layer thickness less than seven times the mean free path (approximately) for conduction across the film, and less than 4.5 times the mean free path for conduction along the film. Henceforth, a method for analysis of microscale conduction is the use of the Boltzmann transport equation (BTE), though conventionally this was used for analysis of gases.

6 Boltzmann Transport Equation

The BTE uses a combination of classical mechanics and quantum approach (Cercignani, 1988).

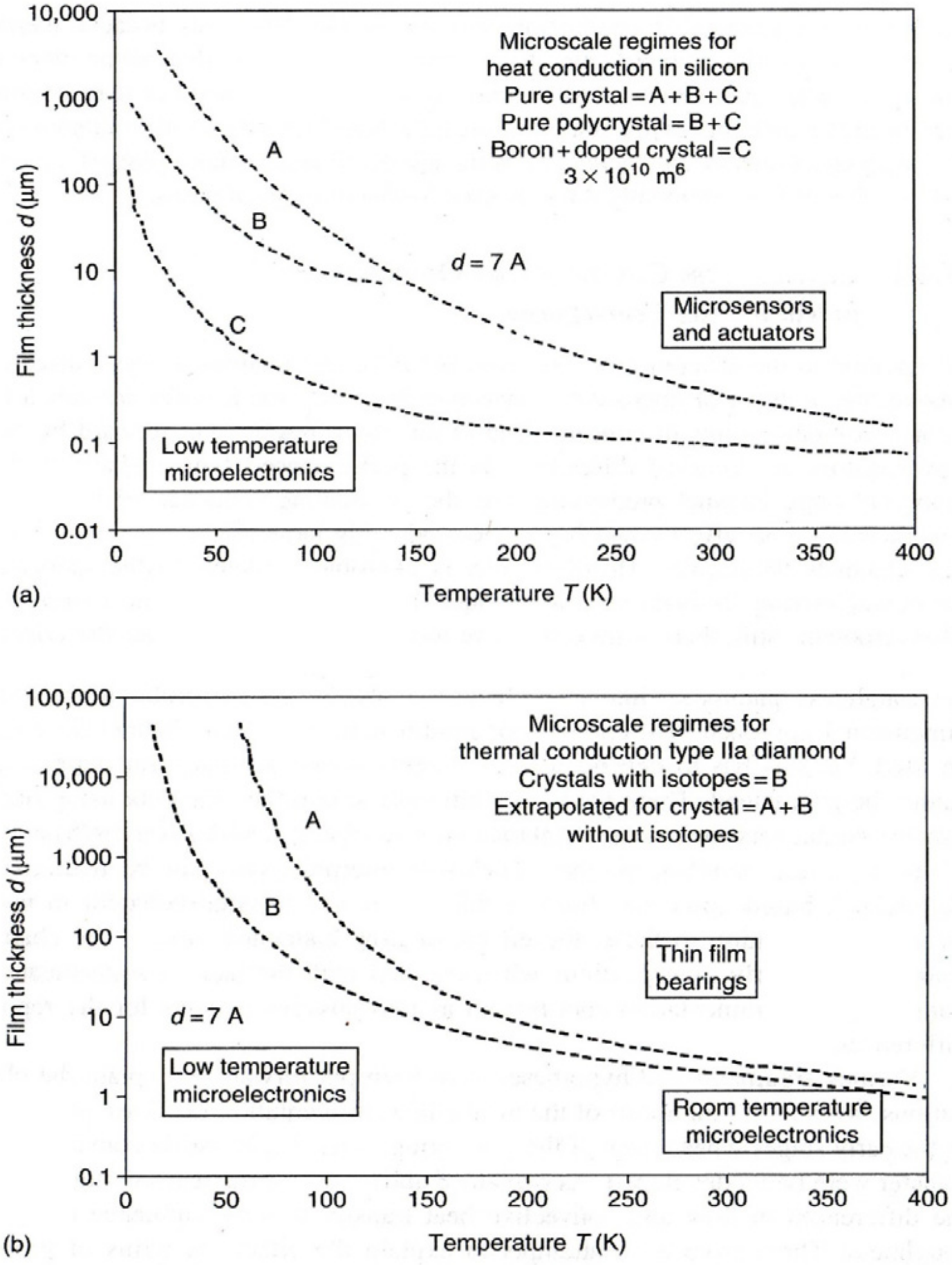

Fig. 5. (a) Microscale regime maps for thermal conduction in silicon films; (b) Microscale regime maps for thermal conduction in diamond films.

Newtonian mechanics is used to describe the classical motion of particles, while the dissipative effects are accounted by quantum mechanical (statistical) considerations, including scattering rates and probabilistic distributions of particles. Thus the equation becomes especially useful while analyzing energy transport from a microscopic viewpoint, in systems where gradients of thermodynamic properties such as density and temperature exists, producing bulk effects. Analysis based on this method can be used to study physical phenomena, as well as to obtain transport coefficients. Further, the formulation is based on a nonequlibrium approach, and the equilibrium state is special case of the general formulation.

The BTE is given by (Cercignani 1988):

$$\frac{\partial f}{\partial t} + v.\nabla_r \, f + F.\nabla p \, f = \left(\frac{\partial f}{\partial t}\right)_{sc} \tag{2}$$

where f being the distribution function of the particles, expressed as f = f(r,p,t) which indicates the probability of occupation of particles with momentum p at location r and time t, and to simplify the equation a relaxation time approximation is often applied to the right-hand side of the equation, which describes the scattering process such as

$$\left(\frac{\partial f}{\partial t}\right)_{sc} = \frac{f_0 - f}{\tau(r,p)} \tag{3}$$

Now f_o being the equilibrium distribution and τ is the relaxation time. On the other hand, appropriate distributions could be Fermi-Dirac distributions for electrons and Bose-Einstein distribution for phonons.

7 Electromagnetic Waves and Maxwell’s Equation

The theory of propagation of energy as electromagnetic waves is also useful in analyzing microscale heat transport. The analysis of radiative transfer, to a large extent, is based on electromagnetic wave propagation. Combinations of the theory with particle theory for energy transport in media by simultaneous solution of the basic equations for the electromagnetic field along with the Boltzmann equation have also been used to analyze microscale energy transport related to radiation heating in thin film (Kumar and Mitra 1999). On the other hand, the Maxwell equation describes the propagation of electromagnetic waves, in a mathematical form in electric and magnetic fields (Kong, 1990; Born and Wolf, 1999; Jackson, 1999; Zhang et al., 2003):

$$\nabla \times E = -\frac{\partial(\mu H)}{\partial t} \tag{4}$$

$$\nabla \times H = \sigma E + \frac{\partial(\varepsilon E)}{\partial t} \tag{5}$$

$$\nabla.(\varepsilon E) = \rho \tag{6}$$

$$\nabla.(\mu H) = 0 \tag{7}$$

Where H, E, ε, μ, ρ and σ represents the magnetic field vector, electric field vector, permittivity, electric charge density and electrical conductivity respectively, (Zhang et al. 2003). But, the general form can be obtained from this set of equations

$$\nabla^2 E = \mu\sigma \frac{\partial E}{\partial t} + \mu\varepsilon \frac{\partial^2 E}{\partial t^2} \qquad (8)$$

And for a dielectric medium the equation becomes (Zhang et al 2003):

$$\nabla^2 E = \mu\varepsilon \frac{\partial^2 E}{\partial t^2} \qquad (9)$$

The solution of the equation for a monochromatic wave can be shown to be

$$E = E_+ e^{-i(wt - q.r)} \qquad (10)$$

Where E_+ being the complex vector, ω being the angular velocity and q the wave vector giving the direction of propagation (Zhang et al., 2003). Furthermore, the vector product $(E \times H)$ is the Poynting vector is related to the power flux of the electromagnetic field

$$S = \frac{1}{2} \mathrm{Re}\left(E \times H^*\right) \qquad (11)$$

An absorption coefficient is derived by combining these exponential attenuation with Maxwell equations, such that

$$S_x = \frac{n}{2\mu c_0} E_0^2 e^{-2q_{im}x} = \frac{n}{2\mu c_0} E_0^2 e^{-a_\lambda x} \qquad (12)$$

Where the absorption coefficient is $a_\lambda = 4\pi\kappa / \lambda_0$ (Zhang et al., 2003).

Another theory investigated to describe radiation phenomenon is the particle theory, where radiation is considered as a collection of quantized energy packets, each having an energy given by

$$E = h\nu = \frac{h}{2\pi} \omega \qquad (13)$$

Where h being the Planck's constant

8 Conclusion and Future Research Works

In this paper we have come into conclusion that

(1) while the present trends of microscale heat transfer studies tend to lead to the analysis and design of heat and fluid flow system at even smaller length scales as nanoscale, the

validity of continuum models and results applying flux laws have yet to be investigated on, to determine the dimensional limits of applications of such results.

(2) Under such conditions, nonconventional modeling based on molecular dynamics consideration would have to be utilized to better incorporate the problem physics at small length and timescales.

(3) In situ local measurements in the channels using flow visualization studies have thrown more light on the physical processes taking place inside small channels, compared to the overall measurements that were possible in the early stages of microchannel research.

(4) Single-phase flow in microchannels has been investigated more extensively than two-phase flow and the conclusions indicate that conventional modeling is adequate.

(5) On the other hand, this is an area that presents a number of considerable challenges in instrumentation, optical, nonintrusive methods could also be utilized in these measurements. But, extension of microchannel research into areas such as biomedical engineering, MEMS, and nanotechnology also seems to be increasing importance and relevance.

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