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# The Transition to Equilibrium in a System with Gravitationally Interacting Particles. I. Temperature Relaxation

**A. M. Boichenko[1*] and M. S. Klenovskii[2]**

*[1]Institute of Fundamental Problems in Theoretical Physics and Mathematics, Moscow, Russia.*
*[2]Institute of Electron Physics, National Academy of Sciences of Ukraine, Uzhgorod, Ukraine.*

***Authors' contributions***

*This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.*

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## ABSTRACT

The distribution function of systems in equilibrium must have the canonical form of the Gibbs distribution. To substantiate this behavior of systems, attempts have been made for more than 100 years to involve their mechanical behavior. In other words, it seems that a huge number of particles of the medium as a result of interaction with each other according to dynamic laws, is able to explain the statistical behavior of systems during their transition to equilibrium. Modeling of gravitationally interacting particles is carried out and it is shown that in this case, the distribution function does not evolve to the canonical form. Earlier, the same results were obtained for classical Coulomb plasma. On the other hand, such a statistical effect as relaxation is well described by the dynamic behavior of the system, and the simulation data are in agreement with the known theoretical results obtained in various statistical approaches.

*Keywords: Equilibrium; gravitationally; interacting particles; entropy; cosmological expansion.*

**Corresponding author: E-mail: boichenko.ifptpm@gmail.com;*

# 1. INTRODUCTION

## 1.1 Status of the Problem and its Current State

The formulation of the dynamical laws of macroscopic bodies, given by I. Newton, led to successes in the quantitative description of their behavior. The discovery of the planets of the solar system Neptune (according to the calculations of W. J. Le Verrier and D. C. Adams) and Pluto (according to the calculations of P. Lovell and W.G. Pickering) are a clear confirmation of this. We can predict the positions of the planets for centuries. Methods for describing the mechanical behavior of systems are constantly being improved [1-12]. However, with an increase in the number of bodies in the system, the prediction of their behavior becomes much more difficult. For systems with a huge number of particles, for example, for gases, liquids, and solids, such a prediction becomes simply impossible. In this case, statistical methods of description are used. But we, on the one hand, understand that this huge number of particles must still be described by the laws of dynamics. On the other hand, when trying to describe this, we are faced with problems that at first glance should not arise. The questions of substantiating the statistical description and their connection with the dynamic description are already becoming essential and come to the fore in science since the beginning of the twentieth century [13].

If a closed system at a certain moment of time is in a non-equilibrium macroscopic state, then the most probable consequence at subsequent moments of time will be a monotonic increase in the entropy of the system. This is the so-called law of increasing entropy or the second law of thermodynamics. It was discovered by R. Clausius, and its statistical foundation was given by L. Boltzmann [14-16].

*Systems in equilibrium.* Conclusions about the increase in entropy in a closed system and the form of the distribution function of systems in equilibrium can be traced in many available monographs. The conclusion that entropy increases (or at least does not change) during an irreversible transition from one equilibrium state to another is proved, for example, in [14] using the postulate that the second kind perpetuum mobile is impossible. The derivation of the distribution function for closed systems in equilibrium, based on the microcanonical distribution, is contained, for example, in [15]. The distribution function $w(E)$ of systems with energy $E$ in equilibrium depends exponentially on the entropy of the system $S$,

$$w(E) \propto \exp\left(S(E)\right)$$

which is why it is stated that the entropy of a closed system in a state of complete statistical equilibrium has the largest possible value (for a given energy of the system). The form of the distribution function of a system as a function of its entropy is rarely used. Most often, depending on the consideration being carried out, its equivalent representations are used either in the form of a Gibbs distribution (canonical distribution).

$$w(E) \propto \exp\left(-E/T\right) \tag{1}$$

where $T$ – temperature of system, or in the form of a Boltzmann distribution. It is generally assumed that when establishing an equilibrium in the energy regions, where the interaction of particles with other particles can be ignored, or the interaction of a particle with only the nearest particle can be taken into account, the Gibbs distribution function transforms into the Boltzmann distribution function of particles.

The use of the above-mentioned equilibrium distribution functions has been carried out in the consideration of a huge variety of practical problems, so there is no doubt that they are confirmed by all our daily observations. But not everything is as simple as it seems at first glance. The fact is that when considering the world around us as a whole, it is not impossible to notice that along with the destructurization of some systems, which just corresponds to the growth of the entropy of such systems, an organization or ordering of other systems takes place. The emergence of man is a vivid confirmation of this. However, the processes of structuring and complicating systems lead not to an increase, but to a decrease in entropy. To a certain extent, these emerging difficulties have not yet been overcome.

*A giant fluctuation.* In principle, this contradiction could be eliminated by assuming that the Universe as a whole is an equilibrium system and that the part of it we observe is a giant fluctuation (the Boltzmann fluctuation hypothesis). The probability of such an event is extremely small (see, for example, [15, 17, 18]). Why, then, do we observe it? The answer to this

question could be found in the anthropic principle. Yes, other configurations of the Universe are more likely, but we would not be able to appear there, because we are the product of this fluctuation, and therefore we cannot observe such configurations. But even this explanation is not satisfactory. The fact is that if the considered fluctuation occurred not in the visible part of our Universe, but only within the solar system alone, then this would already be enough for the appearance of a man, but the probability of such an event would be significantly higher [15].

But the point here is not even that such probabilities are negligible. It would still be possible to somehow come to terms with this, i.e., with the fact that such a fluctuation is very rare. The fact is that in this case, our bodies and brains would have to appear completely formed out of chaos with a reserve of memory, knowledge, and skills although the past that reflects this would never really exist [17]. But our development, on the contrary, is based on information in which we are confident because it reflects the real past in our memory with records of events, experiments, etc., on the basis of which our formation, development, and progress in understanding the structure of our world take place.

But let's say that even this happened, i.e. that our bodies and brains would have to appear completely formed out of chaos with a reserve of memory, knowledge, and skills. What should happen next? And then there should still be a transition to disorder. However, this also contradicts our knowledge. Looking at the metabolic function of even the simplest cells, we see that it involves several thousand interrelated chemical reactions and, therefore, requires a fine mechanism for their coordination and control [19]. This requires an extremely fine and complex functional organization. Metabolic reactions require specific catalysts-enzymes. These are giant molecules with spatial organization, and the body must be able to synthesize them. The biological order is both architectural and functional. Furthermore, at the cellular and supracellular levels, biological order manifests itself in a series of structures and interrelated functions of increasing complexity and hierarchical ordering. Such an organization cannot be the result of a transition to molecular disorder. Therefore, now the explanation through a giant fluctuation is not seriously considered by anyone.

*Open systems.* Since the growth of entropy must occur in closed systems, the explanation of the existence of highly organized structures could be linked to their openness. In fact, it is a way of moving the center of consideration of the issue from the system to the environment surrounding it. To realization such a way, for example, gravity is involved. Gravitational fields play an essential role in the ordering of matter. It is thanks to them that stars, planetary systems, and galaxies arise. In general, the metric tensor depends on time, and external conditions are not stationary. But the gravitational field cannot be included in a closed system since in this case the conservation laws, which are the basis of statistics, would turn into identities [15].

A slightly different aspect related to gravity is noted in [17]. The gravitational interaction leads to the self-organization of matter into galaxies and stars, but taking into account the heat loss to the external environment that accompanies such processes leads to an increase in the total entropy.

Another aspect. For example, when parents clean the children's room with randomly scattered toys, it would seem that the entropy is clearly reduced. However, such actions are accompanied by the splitting of food in the parents' organisms and the transfer of energy to the environment, which ultimately leads to an increase in the total entropy [17].

But, on the other hand, as we move to larger and larger systems, to the limit of the entire Universe as a whole, we must obtain closed systems with increasing accuracy. Therefore, it is not far-sighted to transfer the source of entropy growth that compensates for the processes of self-organization of the systems under consideration to the environment, because a similar question arises about the source of self-organization in the environment itself [18].

*Becoming a problem.* The study of issues related to the general principles of relaxation of statistical systems comes to the agenda as a result of success in the application of statistical and mechanical methods to the description of environmental phenomena. These questions arose together with the task of the so-called "substantiation of statistics", i.e., establishing the connection between physical statistics and mechanics [20-28]. First, there are the difficulties of introducing into mechanics the probabilistic representations that form the backbone of

statistical physics, namely, the connection of the irreversible transition of the statistical system to equilibrium and the reversibility of the equations of mechanics. Secondly, these are the difficulties arising when it is necessary to give a mechanical characterization of systems described by statistical methods. More details about the nature and overcoming of these difficulties can be found in the works [13, 29-53].

The problem of irreversibility has become a serious stumbling block since the works of J.W. Gibbs [28]. He constructed a theory of statistical mechanics based on the time-symmetric (i.e., reversible) Hamilton and Liouville equations. Considering his theory correct and applying it to time-irreversible processes, he comes to mutually exclusive effects. On the one hand, the incompressibility of the phase fluid of a mechanical system follows from the Hamilton and Liouville equations. On the other hand, irreversible processes must proceed with an increase in entropy, while an increase in the entropy of a mechanical system means an increase in the volume of its phase fluid. He associated the growth of the phase volume with the mixing of the phase trajectories of the mechanical system. But doing so, he cancels the Hamiltonian mechanics on which his consideration is based since no increase in the volume of the phase fluid should occur in Hamiltonian systems. To describe the growth of the phase volume, he introduces a "coarse" phase fluid, the growth of the envelope volume of which is intended to be responsible for the growth of entropy. At the same time, for a finite mixing time, the phase cells of the "coarse" phase liquid must be taken of finite sizes. However, another problem arises here. On the one hand, as the final size of the cells decreases, we must get a better-described effect of irreversibility, but it turns out the opposite: the systems become more and more reversible. On the other hand, if the size of the finite cells is immediately directed to zero, then the mixing time tends to infinity, i.e., the effect that we would like to describe disappears. In addition, relaxation to equilibrium falls out of consideration in this approach, and we immediately get the equilibrium state of the system.

The first works on the "substantiation of statistics" were associated with the substantiation of the ergodic hypothesis, which was highlighted by the review [13]. According to this review, Boltzmann and Maxwell argued that the microcanonical distribution is valid when the system is ergodic [18]. In the works of Boltzmann and Maxwell, however, one cannot find such a rigid statement, and it is, according to [18], on the conscience of the Ehrenfests. Imperceptibly attributing to Boltzmann and Maxwell an unnecessarily rigid statement of their own, the Ehrenfests convinced the bulk of physicists that ergodicity supposedly underlies at the basis of equilibrium statistical physics. It is now known that this is not the case. A statistical system can be non-ergodic, an ergodic or quasi-ergodic motion can be non-stochastic, and the phase volume of a statistical system does not have to be a single whole [18].

*Ergodicity, metric transitivity.* Clarification of the question of what the ergodic hypothesis is in mathematical language began with the works [30, 53-58]. The problem was reformulated in terms of the volume of the phase space and the measure of the phase set. It turned out that the classes of ergodic systems coincide with the classes of metrically transitive systems. However, it cannot be said that the understanding of this issue has significantly advanced. Birkoff's theorem, according to [33], only replaces one (ergodic) hypothesis with another (the metric transitivity hypothesis), which is no less dark and mysterious. In addition, it is difficult to prove, for example, by the form of the Hamiltonian of the system, whether the motion of the system is metrically transitive or not. Even examples admitting the solution to this question are extremely rare [34]. The research of Birkhoff and von Neumann was continued in the works [32, 33, 36, 59-68]. Gradually it became clear that the ergodic hypothesis significantly narrows the class of systems that obey it. More and more examples of non-ergodic statistical systems appeared.

Von Neumann has already shown [30] that quantum ergodic systems are not degenerate. This result greatly limits the possibilities of the ergodic approach, since most of the quantum systems of interest are degenerate [19]. Even an ideal gas turned out to be a non-ergodic system, since the phase trajectory describing it does not fill the energy surface tightly, being constrained by $N$ isolating integrals, which are the energies of each of its $N$ constituent particles [69]. It also turned out that ergodic (and quasi-ergodic) motion can be non-stochastic [70]. It was shown that for a wide class of interacting dynamical systems, it is possible to construct periodic orbits lying in a certain subspace (on invariant tori) of the ergodic surface [18, 19, 69, 70].

*Mixing.* The fact that the ergodic hypothesis cannot be used as a basis for statistical description was clearly recognized by N.S. Krylov, who began the development of the Soviet ergodic school (A.N. Kolmogorov, Ya.G. Sinai, etc.). This hypothesis, according to him, is both insufficient and not necessary for statistics [29]. Instead, he puts forward the mixing and conducts a consideration of systems with mixing. However, since systems with mixing are systems of the ergodic type [29], and the ergodic motion may not be stochastic (see the previous paragraph), then this deprives the mixing of the advantages for which the transition from ergodicity to mixing takes place [18]. In addition, the roughening procedure associated with the consideration of irreversible mixing is of an operational nature. The different accuracy of determining the relaxation time and the type of its measurement dictate a different division of the phase space into the regions under consideration, and, consequently, each choice of the type of measurement corresponds to its own relaxation time [29]. Thus, the attempt to quantify the relaxation time as a mixing process did not lead to success. In fact, Krylov comes to the conclusion that it is impossible to justify statistical mechanics within the limits of classical representations and places his hopes on quantum representations [18].

The development of Krylov's works was the use of the set theory apparatus to describe the mixing of the phase density [71, 72], which led to the concepts of dynamic or metric entropy (in other words, Kolmogorov entropy or K-entropy). Systems for which K-entropy is constant and positive have been called K-systems. Their main property turned out to be K-mixing, which is associated with exponential local instability. The K-mixing scheme is based on above mentioned Gibbs mixing scheme, where its fundamental flaw it also noted. In addition, it should be noted that being an irreversible, K-system, with its time-constant rate of growth of physical entropy, cannot be identified with the irreversible system traditionally considered in non-equilibrium statistical physics, since in the latter case its relaxation ends in an equilibrium state.

*Integrability.* An important characteristic of the system is its integrability, i.e., the presence of analytical integrals of motion and, accordingly, solutions in an analytical form. One of the most studied examples is the three-body problem. Numerous attempts were made to reduce this problem to an integrable one, but at the end of the XIX century, E.H. Bruns and A. Poincare proved that this is impossible. It turned out that most of the interesting problems of classical dynamics, starting with the three-body problem mentioned above, are not reduced to integrable systems. With the exception of some special cases, the Hamilton equations do not admit any other analytic single-valued integrals other than the energy integral. In addition, if we initially take an integrable system and perturb it with even an infinitesimal perturbation, then in the general case we end up with a non-integrable system. The situation in which a dynamical system does not have integrals of motion and, as a result, a continuous change in external conditions causes a jump in the behavior of the system, looks, from the point of view of classical mechanics, catastrophic (the so-called Poincare catastrophe) and is the subject of consideration of the theory of catastrophes. Apparently, due to this circumstance, Poincare identified systems with non-analytic integrals of motion with stochastic ones. However, we now know that this is not the case. The system can be non-integrable (KAM-theorem), but at the same time make regular movements. The value of this extremely important Poincare result is reduced by the fact that there are still no general methods for studying systems for integrability or non-integrability. In addition, stochasticity was associated not so much with the non-analyticity of the integrals of motion, but rather with their non-isolating nature.

*Non-integrability: regularity.* The possibility of regular motion in the non-integrable case is demonstrated in the Kolmogorov-Arnold-Moser(KAM)-theorem [73-76]. If the Hamiltonian of the integrable system is not degenerate and the ratio of the frequencies of motion of the system in the action-angle variables is expressed by irrational numbers, then the motion of the system occurs on non-resonant tori. When a sufficiently general perturbation is imposed away from the resonance of the unperturbed system, the system becomes non-integrable; however, the motion of the perturbed system for most initial conditions occurs on nonresonant invariant tori that differ only slightly from the perturbed ones. Invariant tori, on which the quasi-ergodic motion of integrable systems occurs, are destroyed under perturbation when a non-integrable additive is added to the integrable Hamiltonian, but not all at once. The set of tori that remain invariant under such a perturbation has a finite measure that tends to zero with the growth of the perturbation. This means that the

system can have non-analytic integrals of motion and be non-integrable, while still remaining non-stochastic, i.e., performing a quasi-periodic motion, which is also quasi-ergodic.

*Non-integrability: regularity and stochasticity.* The possibility of both regular and stochastic movements in the system at the same time, starting from [77], has been demonstrated in a variety of numerical experiments [38, 39, 41, 50, 78-81]. The Hamiltonian of systems in such numerical experiments is chosen as the sum of the unperturbed Hamiltonian for the integrable system and some perturbation. Even in the case of the system with two degrees of freedom, considered in [77], stochastic regions are separated from each other by regions with regular trajectories. Global stochasticity for systems with two or more degrees of freedom, when the entire phase volume of the system is a single stochastic region, occurs only with a sufficiently large perturbation. In the general case, the regions of stochastic motion occupy only a certain part of the phase space.

*Instability.* One of the key points in the transition of a system to stochastic motion is the instability of the motion of its particles. Poincare was the first to draw attention to the connection between the stochastic motion of a mechanical system and the instability of such a motion, using examples of the unstable equilibrium of a cone standing on its top, problems of meteorology, and the distribution of small planets in the zodiac. An absolutely insignificant cause, which escapes us because of its smallness, he writes, causes a significant effect [82]. Or another example. Consider the motion of gas molecules. As the weakest external disturbance, we take the gravitational interaction of a molecule of this gas with a proton at the other end of the Universe, i.e. at distances of about $10^{10}$ light years. Even such an insignificant interaction completely changes the motion of the molecule in just ~ 60 collisions with other gas molecules (under normal conditions), i.e., in the order of $10^{-8}$ seconds [81].

The instability of the particle motion is characterized by Lyapunov exponents. Accordingly, irreversible stochasticity became associated with the local instability of the system's phase trajectories. In a large number of works, the divergence of phase trajectories $D(t)$ was considered on the basis of the instability increment $a$

$$D(t) = D(0)\exp(at)$$

At the same time, it was automatically assumed that the divergence of the trajectories would be irreversible. At this point, the system described by symmetric equations in time imposed a further asymmetry of its behavior. The correct one should recognize the dependence (see [18] for details)

$$D(t) = \frac{1}{2} D(0)(\exp(at) + \exp(-at))$$

In fact, this is where the confusion in the consideration of reversible and irreversible dynamic chaos was born since it was in this place that the system was imposed with properties that it does not possess. The inaccuracy of the conclusions of many works was reduced to this (see, for example, [29, 62, 63, 83-86]). So, for example, when studying the divergence of the trajectories of elastically interacting balls, Krylov concludes [29] that their trajectories inevitably run up, forgetting that, for example, if we let the balls go in the opposite direction, then his consideration should lead to the convergence of the trajectories. Consideration of the works [85, 86] is based on the idea borrowed from Krylov that the reflection of a beam of rays from a convex surface is necessarily scattering in nature.

The stochasticity of movement in statistics implies an irreversible transition to chaos. In the case of reversible dynamic chaos, such a transition is impossible. If, for example, the gas originally described by symmetric time equations is located in the corner of the vessel, then it will not occupy the entire vessel over time, as is commonly assumed. If this happens, it will be for an extremely short time. The gas will constantly change the shape of its filling of the vessel, then occupying its entire volume, then some parts of it, again and again returning to the initial corner [18]. Dynamic chaos theory studies dynamic chaos in reversible mechanical systems described by time-symmetric equations. With respect to a reversible system, we can only talk about maintaining dynamic chaos, it cannot arise in such a system, since this would be associated with an irreversible increase in entropy [18].

*Instability and blurring of the initial state.* So, although a small change in the external conditions in the case of instability of motion leads to a jump in the behavior of the system, but the irreversibility will not occur. Note that Poincare, when he discusses the roots of

randomness in mechanics [82], the instability of a dynamical system, due to which a small cause generates large consequences, always inseparably appears with the blurring of the initial state (for more information, see the discussion below). Only the instability of the motion of the system's particles is not sufficient for the transition to irreversible chaos.

*Quantum mechanics.* So, the question of the irreversibility of processes in the transition to thermodynamic equilibrium remains unanswered. Some researchers solve this problem by going beyond just the classical dynamic description. It is assumed that there is a quantum component in the description of any real system [15, 17, 29]. At the same time, the quantum equations themselves are also time-reversible and in this respect do not help to explain the irreversibility of the processes occurring during thermalization. For irreversibility, the interaction of the classical and quantum subsystems is necessary. So, the probability of the result of process *B* can depend on the results of process *A* only if process *A* took place before process *B* [15].

If we take the problem more broadly, then in fact irreversibility in quantum mechanics is closely related to the problem of quantum measurement. Even if we do not specifically measure anything with some measuring device, the macroscopic environment willy-nilly constantly acts as a continuous meter since it constantly interacts with quantum objects. The Schrodinger equation is time-reversible and describes the wave function of the system, which gives the probability of finding the system in a particular state. In accordance with the probabilistic description, the particle can be here and there, but the experimenters never see this. They always find a particle definitely in one position or another, and never see it partly here and partly there. During the measurement, the so-called collapse of the wave function occurs, and only one of the many possible outcomes is realized [17]. The Schrodinger equation itself does not regulate this collapse in any way. We need equations describing this collapse, as well as equations describing the development of decoherence in the system [17, 87]. The solution to the problem of the collapse of the wave function and decoherence could provide an answer to the question of the reversibility of the evolution of systems. Note also that if a collapse of the wave function occurs, i.e. there is a measurement in the system, then the system itself changes. After the measurement, we have a completely different system, which is partially degraded [88].

However, whatever the answer is, sooner or later we will still run into the second principle of thermodynamics. Eggs falling on the floor should break, a piece of ice immersed in a warm bath should melt, and not vice versa: broken eggs will not collect in whole, and a piece of ice will not grow, etc. The direction of time in one way or another seems to be related to the evolution of the entropy of the system.

Another aspect of the possible influence of quantization on the initiation of irreversibility in real systems can be traced in the cycle of works on modeling the classical Coulomb plasma (see [89-93] and references therein). We will discuss this cycle of work in detail below. Now we note that in the above-mentioned works, the particle energy distribution function did not transform over time to the Boltzmann form at large magnitude negative energies, which was completely consistent with the well-known entropy conservation theorems. As a result, the plasma "refused" to recombine. Theoretically, it was possible to describe this behavior at the cost of abandoning the ratio of the detailed balance to the coefficients of diffusion and electron mobility. A different relation was proposed for the diffusion and mobility coefficients of the microfield nature, resulting in a powerful upward drift of electrons along the energy axis, which prevents the establishment of the Boltzmann distribution function. Why are these effects not observed in real experiments? In particular, the shock recombination process is well known in plasma, the rate constant of which is proportional to the electron temperature to the degree of "–9/2" (Thomson recombination). The explanation was given on the basis of the need to take into account quantum effects. Quantization will not prevent the proposed explanation when the characteristic microfield jump exceeds the characteristic level difference in the proton + electron system ($\propto 1/n^2$, where $n$ is the main quantum number), i.e., for relatively large $n$. When this ceases to be the case, the ratio of the mobility and diffusion coefficients of the microfield (multiparticle) nature should pass in this energy region to the two-particle ratio of the mobility and diffusion coefficients in accordance with the ratio of the detailed balance, which will cause recombination.

In the parameter domain, where this effect can be well recorded, the recombination delay time is only a few times longer than the usual recombination time, so the effect under discussion cannot be unambiguously interpreted.

On going over to ion-ion plasma, the difference in recombination times becomes significant, however, the creation of such a plasma is still problematic [94]. The problem is that the negatively charged ion should retain its excess electron for quite a long time. According to [95], the destruction of negative ions can be prevented by the solvate shell of these ions. If this turns out to be true, then the predicted delay in recombination may find its natural explanation. In particular, in this case, the recombination time of such an ion-ion plasma may already be sufficient to explain the nature of ball lightning and its lifetime (on the order of a minute).

That is, in this case, the quantumness provokes the transition of the system to traditionally understood equilibrium, which in a purely classical approach would not take place. Recombination begins and, accordingly, the distribution function takes on a recombination form.

*Quantum statistics*. When involving quantum mechanics in the explanation of irreversibility and in the attempt to explain statistical laws through dynamical ones, it should be borne in mind that in the quantum case we will deal with quantum statistics, which is fundamentally different from the classical one. This is expressed in the fact that in the classical case we deal with a local description and in the quantum case with a non-local one [96, 97]. But these approaches are incompatible, which is expressed in the absence of hidden parameters in quantum systems [97].

De Broglie's attempts to circumvent the absence of hidden parameters in quantum systems were considered in [97]. It can be said that de Broglie's attempt of the 1950s was the last attempt at such an explanation, in which, as he believed, it would be possible to have hidden parameters in his previously advanced theory of the double solution applied to the explanation of the corpuscular-wave duality in the local approach. The problem of the presence of hidden parameters in the system reappears in connection with the Bell inequalities. It turns out that this problem has been known in mathematics for a very long time.

The problem of the possibility of simultaneous realization of *n* random variables with given correlations among themselves, when these quantities are independent of each other but depend on some other random variables, arises within the framework of the classical probability theory long before the appearance of Bell inequalities (1964). The necessary conditions for such an implementation were originally obtained by J. Boole in the form of inequalities about 100 years before the introduction of Bell's inequalities, and the general solution for a system of *n* random variables was obtained by N.N. Vorobyov in 1962 [98].

In the case of two particles, Bell's inequalities, which connect three random variables that can take values $\pm 1$, independent of each other but depend on some random variable $\lambda$, were introduced to analyze the Einstein-Podolsky-Rosen (EPR) mental experiment [99, 100]. Quantum mechanics presupposes a probabilistic description (see above), so if Bell's inequalities are satisfied for non-commuting observables, then this leaves hope for the presence of hidden parameters $\lambda$. In other words, quantum probabilities and uncertainties could be reduced in this case to the classical theory of probability, and the state of the system, thus, would be specified not by a wave function, but by a set of hidden parameters.

However, consideration of Bell's inequalities as applied to the wave function of two particles with zero total moment (EPR states) shows that by choosing the directions of the axes on which the projections of angular moments or particle spins are measured, the Bell`s inequalities can be violeted [101-103] for the correlators of the projections on the given axis [98, 104]. This violation is known as the EPR paradox. However, this behavior of quantum correlated systems is paradoxical only from the classical local point of view. If we assume the presence of non-locality, then we have no paradox.

The generalization of Bell's inequalities in the case of three particles is carried out by D. Greenberger, M. Horn, A. Zeilinger (1989), starting from which a new stage in the study of non-locality in many-particle systems opens [104].

Thus, even if quantum mechanics gives us the key to describing the irreversibility of phenomena in time, it is unclear whether it will be able to cope with embedding in the backbone of

classical statistics, which is responsible for explaining the observed phenomena.

*Synergetics.* Oddly enough, some clarification of the issues under consideration occurred as a result of the development of synergy. The equations of synergetics are not necessarily mechanical (and not even necessarily physical). They may relate, in addition to physical, and other various disciplines, for example, chemical, biological, social, etc., but they are dynamic in the broad sense of the word. The number of considered synergetic equations is constantly growing. Many of these equations have their own names: the Turing, Lorentz, Cook-Roberts, Brusselator, Oregonator, autowave, Eigen-Schuster, sine-Gordon, Schlegel, Roesler, Volterra equations on the competition of biological species, etc. (see [18, 105-108] and references therein). In most cases, these equations can be represented as

$$\frac{dx_i}{dt} = V_i(\vec{x})$$

where $x_i$ and $V_i$ are the some characteristics of object $i$, and therefore, the values of $x_i(t)$ can already be interpreted as microscopic coordinates (and impulses) of a dynamical system. However, the fundamental difference between the equations of synergetics and reversible mechanical equations, which are used to justify statistical laws, is that they are not symmetric in time and, in their overwhelming majority, are not Hamiltonian. Recall that for Hamiltonian systems, Liouville's theorem for the density $\rho$ of a phase liquid is valid

$$\frac{d\rho(q,p,t)}{dt} = 0 \qquad (2)$$

where $q$, $p$ are the generalized coordinates and impulses of the system. But if the equations of synergetics are time-irreversible and not Hamiltonian, how can this help us when considering time-reversible Hamiltonian systems?

Since the equations of synergetics are not Hamiltonian, they allow for a change in the phase density, and, consequently, in the entropy of the system. But at the same time, we come to the conclusion that when describing irreversible processes, the kinetic and dynamic approaches are not compatible with each other. The dynamic approach postulates the independence of the degrees of freedom of the system from each other, for which, as a result, it is possible to write separate dynamic equations of motion. In the kinetic approach, the condition

$$\frac{d\rho(q,p,t)}{dt} \neq 0 \qquad (3)$$

can only mean the existence of a statistical relationship between the time functions of $q_i(t)$. The growth of entropy in the irreversible case in the kinetic approach leads to the fact that the phase fluid of such a system must occupy an increasing volume (3) of the phase space but must remain incompressible (2) in the dynamic approach. Therefore, when describing such a situation in the dynamic approach, the phase trajectory is forced to break at each point with the formation of a discontinuous fractal structure everywhere [18]. That is why the phase structure of synergetic systems contains strange attractors, whereas the phase structure of Hamiltonian systems cannot contain such attractors [18]. The closer the fractal dimension is to the topological one, the closer the system is to equilibrium. Thus, when trying to describe the growth of entropy in closed isolated systems in the dynamic approach, we come to the fact that we actually lose the very concept of the trajectory, or, if we still use the dynamic approach, we come to the fact that the trajectories in question must constantly break.

We also note that in irreversible synergetic systems, the course of irreversible ordering processes is the norm, in contrast to the equilibrium states considered by traditional statistical physics.

***Different arrows of time.*** From the previous discussion, it can be seen that the hopes placed on the mechanical description of statistical laws have not yet been realized. Nevertheless, somehow it was necessary to explain the irreversible behavior of systems over time, so in parallel, there was a consideration of other aspects of irreversibility, which was expressed in the introduction of various arrows of time. Let's consider them as the most common in the literature.

*Thermodynamic time scale*: time flows in the direction that contributes to the realization of the greatest number of possible states. In other words, the thermodynamic scale is closely related to the entropy of the system. In this scale, the system tends to move over time to the state

with the maximum statistical weight. I.e., the thermodynamic scale determines the direction of the flow of time in which the disorder grows.

*The cosmological arrow of time* is associated with the expansion of the Universe (see, for example, [109]). This raises a lot of questions. If the Universe begins to collapse, then such a clock should turn "back". But this should not happen. On the one hand, for example, during the flight of a rocket launched from Earth, the change of the expansion phase to the compression phase of the Universe should not affect the movement of the rocket in any way. For the rocket, there is no rearrangement of any physical laws that determine its flight, in particular, the clock placed in the rocket will continue to run monotonously as well [110].

On the other hand, in the compression phase, the Universe will have to return to its initial state, i.e., to a state of complete order, which is incompatible with the life of biological beings. Why should the initial state be characterized by complete orderliness? In the initial state, the laws of physics known to us do not work. It would be more natural to consider the initial state as a state with a complete disorder, however, for further consideration of the behavior of the Universe, we need to know the behavior of the various histories of the behavior of the Universe at the boundary of space-time in the past. In other words, we must know what we do not know and cannot know.

The way out of this situation, according to [111], consists in the absence of space-time boundaries. Thus, the direction of the cosmological arrow of time is determined not by the cosmological expansion but by the condition of no boundaries. The coincidence of the directions of the cosmological and thermodynamic arrows is determined by the fact that the phase of cosmological compression is not observed by living beings. Before the compression phase, the Universe would be in a phase of almost complete disorder, and the disorder could not increase much in the compression phase. To live, you need to consume food, which appears in the form of an ordered form of energy, and convert it into heat, i.e., into an unordered form of energy. Therefore, there can be no intelligent life at the stage of compression [111]. So, when considering the cosmological time scale, we run into the fact that the determining factor for living beings is the thermodynamic time scale.

*The causal arrow of time*: we can influence the future but not the past.

*The psychological arrow of time*: we remember the past but not the future.

This arrow is closely related to the causal arrow of time. The structure and principles of the functioning of the brain are not fully revealed, so the consideration of the psychological arrow of time is carried out in [111] on the example of the computer. The explanation of the direction of a given time arrow is also determined by the thermodynamic arrow. So, for example, to organize the memory of a computer, you need to expend energy for its operation. To make sure that the memory is in an ordered state, you also need to expend energy. The need for cooling (fan operation) also requires energy costs, etc. It can be shown that the increase in disorder will always be greater compared to the increase in the ordering of the computer's memory [111].

*Quantum mechanical arrow of time* [17, 112]: the direction of time is determined by decoherence processes and other quantum-mechanic effects, for example, reduction of the wave function (see above), etc. Note that the processes of decoherence lead to the degradation of the system, i.e. their result is also an increase in its entropy and, thus, the quantum mechanical arrow of time is closely related to the thermodynamic arrow.

So, the defining arrow of time is ultimately the thermodynamic arrow of time. A whole chapter in [17] is devoted to the consideration of the justification of this provision. It should be noted that there are other points of view.

For example, it is noted in [105] that although the concept of entropy and related concepts are extremely useful in the thermodynamics of irreversible processes, they turn out to be too crude when considering self-organizing structures. In general, in such structures, the entropy changes by a very small amount. In addition, it is known from statistical mechanics that entropy can fluctuate and thus other approaches are needed here [105].

The authors [107] believe that using an entropic-negentropic language to analyze the world of non-stationary nonlinear processes seems to mean almost the same as going into the microcosm with an hourglass and a measuring tape. For the nonlinear world, other principles of the direction of the flow of processes, the

principles of the formation, unification and development of structures, the principles of economy and acceleration of evolution are valid. And these principles cannot be achieved by fitting the classical language [107].

The direction of the flow of time from the point of view of [110] is determined by the radiation condition, which is not symmetric with respect to the past and future. The growth of entropy in this case is a consequence; it is formulated after the concept of the future has already been defined. As for the growth of entropy, it is convenient to use this condition to determine the direction of time growth, but in itself, it is a consequence and not a cause of the direction of time events [110].

Then we come to the nature of time as a philosophical category. Is it there? Or is it just a way of describing the world around us, invented by us for the convenience of this description? If so, does it exist by itself or only in conjunction with the material component? Does it have a beginning and an end? Was it born together with the birth of the Universe, i.e. it is its attribute? Is it continuous or discrete? Many of these aspects are covered, for example, in [17, 110, 111, 113-116].

As for the issues raised in this paper, we have returned to the starting point. Entropy should increase over time. The various arrows of time introduced to explain its direction are actually closed at this stage of the development of physics to the thermodynamic arrow of time, i.e., they are determined by the growth of entropy. It turns out a vicious circle. On the one hand, an irreversible transition to an equilibrium state with an increase in entropy must occur over time. On the other hand, the irreversibility of the processes in time (i.e., the arrow of time) is associated with the growth of entropy, which determines the direction of time, i.e. its arrow. As can be seen from the above discussion, an attempt to justify such behavior inherent in the consideration of the statistical description of systems, involving a dynamic description of the same systems, does not lead to success. At the moment, there is no justification for the statistical behavior of systems by their mechanical description.

## 1.2 Equilibrium. What Happens to Entropy?

It would seem quite natural to try to describe the growth of entropy during the transition to equilibrium in statistically considered closed systems through their mechanical description. Such attempts, made for more than a hundred years and described above, did not lead to success. But let's ask ourselves why? Why is it impossible to justify the statistical behavior of systems by considering their mechanical behavior? Or maybe it's just not possible? Generally speaking, there are serious obstacles to this. After all, there are strictly proven theorems on the conservation of entropy in closed systems in both the classical and quantum cases [88, 117, 118].

On the one hand, according to these theorems, the entropy of closed systems should not change but on the other hand, it should increase. Why its growth most often is observed in real systems? In this regard, the cycle of works (see [89-93] and references in them) is of interest. When considering the classical Coulomb plasma by numerical modeling methods, it turned out that the plasma does not "want" to recombine. The equations of motion of charges as classical particles moving in accordance with Newton's equations were solved numerically. When different initial conditions were set, the charge distribution function after some time passed into a stationary state, the form of which did not take the form of Boltzmann distribution in the limit of large magnitude negative energies. The distribution function of the charge system began to follow the basic laws of statistical mechanics only when the stochastiser was "turned on". Stochastisers of various types were used: calculation with different counting errors; introduction of rough walls, resulting in diffuse reflection of particles from them (i.e. with a random direction inside the volume under consideration); introduction of thermostatic walls, when reflection occurred with a random direction of velocity and kinetic energy; permutation stochastisation, when the coordinates of the particles did not change but the directions of motion of the particles were randomly redistributed (the speed of one particle was assigned to another, the speed of another to the third particle, and so on); stochastisation by inelastic collisions with a hypothetical gas, etc. The system reacted to different types of stochastisers with different degrees of intensity.

It turned out that only in the presence of a stochastiser it was possible to obtain a diffusion form for the distribution function, obtained using the principle of detailed equilibrium. The role of the stochastiser, according to [90], was the loss

of dynamic memory by the system. In other words, the stochastiser led to the fact that the trajectories of the charges began to differ significantly from the trajectories obtained under the condition of only the Coulomb interaction of the particles with each other. The degree of impact of the stochastiser on the system depends on its power and specifically on the system itself, and its resistance to changing its dynamic memory. For example, rounding numbers in numerical calculations also plays the role of a stochastiser, however, it was not able to transform the form of the distribution function to the desired form from the point of view of statistical mechanics.

So, what happens in real systems? Apparently, in real systems, there is an uncontrolled impact on it. Let's discuss some possible types of uncontrolled impact on systems. It was noted above that the instability of the trajectories of the particles is not enough for their stochastic motion to occur. According to Poincare, stochasticity will occur in unstable systems only if the initial state is blurred. Generally speaking, if the initial state of the system is blurred, then it is likely to remain blurred at other, subsequent points in time.

What can be the reason for the blurring of the system states at any given time? Such a reason may be the resulting uncertainty of the coordinates and impulses of the particles of the system when it interacts with its environment. Whether we like it or not, when the system interacts with its environment, its parameters are continuously measured. There is no question of an experimenter here. In the experiment, the researcher takes special efforts to assemble the installation, allowing it to interact with the system to get the necessary information about the system from it. In the absence of an experimenter, the system at any given time still has some environment with which it interacts. Thus, in the case of the experimenter, the system is affected by the environment that he has chosen, and in its absence, the system is affected by its own environment. For example, when a quantum system interacts with its classical environment, its wave function will collapse at every moment of time, resulting in (according to the Heisenberg uncertainty relations) the same uncertainty of the state of the system under discussion. As noted above, after the measurement, we have a completely different system, which is partially degraded [88].

The reason for this interaction of the system with its environment may also be the non-locality of our world [96, 97, 119]. It is shown in [97] that the explanation of the corpuscular-wave dualism is naturally obtained in the non-local approach. In this approach, using the string theory apparatus, it is possible to obtain both the wave equation for particles and the Heisenberg uncertainty relations. Thus, by virtue of non-locality, the system knows everything about its environment and takes this into account in its evolution. So, the possible scenario of the system behavior described here leads to the loss, at least, of the system's closure.

The presence of a stochastiser or several stochastisers is another reason for uncontrolled exposure to the system. For example, it is described in [90] that a random periodic permutation of the particle velocities without changing the total energy of the system led to a deformation of the distribution function of the system to a diffusive form. Such a permutation, from the point of view of [90], led to the loss of the dynamic memory of the system. If we use the language of mechanics, then, in this case, there was at least a loss of the Hamiltonian property of the system, and for non-Hamiltonian systems, the entropy can already change, which is fully manifested in the works on synergetics (see above). In addition, such a stochastiser will also lead to a blurring of the state of the system, which is consistent with the consideration of the previous paragraph, i.e., as if to the loss of its closure. Note that the occurrence of the blurring of the state of the system, considered in the previous paragraph, when it interacts with its environment, can in turn also lead to the loss of its Hamiltonian character.

Thus, in real systems, entropy can actually change, and this is possible at least in the examples listed above due to the interaction of the system with its environment and due to the loss of dynamic memory by the system in the presence of various uncontrolled stochastisers. These factors will lead to the loss of the closure and Hamiltonian nature of this system.

If we ignore the factors that bring the particle distribution function to the canonical form in real systems, then the closed system itself, in general, will not be deformed, as a result of the action of the mechanical equations, to the state to which it should pass from the point of view of statistical behavior. In this case, there must be an increase in entropy, which cannot occur. The examples, where both the regular and stochastic components co-exist in the system (KAM-theorem [73-76], the results of [77]) have already

shown this, although the analysis of the distribution function was not carried out directly in them. This was directly demonstrated in the numerical solution of the mechanical equations for a system of Coulomb particles with a simultaneous numerical analysis of the distribution function in this system [89-93].

### 1.3 Why was a Gravitationally Interacting System Chosen for the Simulation?

So far, there is only one example of a detailed numerical study of the behavior of a mechanical system during its transition to equilibrium (see [89-93] and references therein), in which the particle distribution function was analyzed on the example of a classical Coulomb plasma. The consideration of other mechanical systems in connection with the above-mentioned features of the description of their statistical and mechanical behavior is of immediate interest. In this paper, we analyze the numerical behavior of point-like gravitationally interacting particles.

It would seem that we should get approximately the same results since the law of universal gravitation and the Coulomb interaction of particles are functionally the same. In both cases, the interaction force is proportional to the product of the charges (in the gravitational case of the masses) of the particles and inversely proportional to the square of the distance between them. However, this is where the similarity of these systems ends. As for the differences between these systems, it is huge. The differences begin with the fact that there is no equivalent to negative charges, i.e., there are no negative masses in the gravitational interaction. Particles in the gravitational case only attract and never repel, as in the case of electrical interaction. It is believed that electrodynamics is currently the most understood in terms of its description and practical use (television, radio, etc.). Electrodynamics is quantized. The description of electromagnetic effects is possible with great accuracy. For example, the theoretical value of the anomalous magnetic moment of an electron in relativistic quantum electrodynamics coincides with the experimental value with an accuracy of up to 11 significant digits (relative uncertainty of $2\ 10^{-10}$) [120-123].

As for gravity, it is not even quantized [124, 125]. The equality of the gravitational and inertial masses in General Relativity (GR) is considered as an exact law of nature. It is believed that this fact is proven in theory. But in fact, the equality of masses, in theory, is valid only in systems of a special kind, in particular, in Cartesian systems. In other systems, the inertial mass can take an arbitrary positive or negative value [124, 125]. There is also a more key problem of determining the gravitational energy and the laws of conservation of energy-momentum in GR. Great efforts to solve it did not lead to success. The introduction of Einstein's pseudotensor seemed to chart the way to solve it. But in this case, during space-time transformations, the energy of the gravitational field changes (for example, the Bauer paradox). K. Moeller formulates conditions for the pseudotensor of the gravitational field, which should exclude obtaining ridiculous results, but he proves himself the theorem that they cannot be satisfied in principle [124, 125], etc.

The gravitational interaction occupies a special position among other types of interactions. After the failures of gravity quantization, the idea of the secondary nature of the curvature of space-time is put forward [124, 125]. It is assumed that gravity is not a fundamental interaction but is a macroscopic (long-wave) limit of a more general theory due to quantized fields [124, 125]. It is also believed (see, for example, [17]) that the gravitational interaction can lead to a decrease in entropy. Thus, as a result of the Big Bang, such structured objects as stars, galaxies, clusters of galaxies are formed from initially uniformly distributed matter and fields in the Universe. All this determined the choice of the gravitational interaction as the interaction in the simulated mechanical system considered below.

However, despite the noted differences, the electromagnetic and gravitational interactions, nevertheless, should be closely interconnected. Since 1955, where gravitational electromagnetic entities was pointed [135] up to now where interconnections of quantum light theory, quantum field theory and gravitational-electromagnetic equation [136, 137] are investigated, these questions are continuously under consideration.

In addition, various effects and processes are of particular interest, the theoretical description of which is well known at the statistical level. Using the example of gravitationally interacting particles, the behavior of these particles over time will be modeled based on the solution of Newton's equations of motion. A comparative analysis of the characteristics of the various processes in the simulated particle system will be

carried out to find out whether well-known statistical effects will actually manifest themselves in the system under consideration. In this paper, the discussion will focus on relaxation processes.

## 2. METHODOLOGY

### 2.1 Modeling the Dynamics of Gravitationally Interacting Particles

The simulation of a gravitationally interacting system of particles consisted in the numerical solution of the equations of motion inside a cube. The system consisted of 300 identical particles gravitationally interacting with each other. At the initial moment of time, the coordinates and velocities of the particles were set using a pseudorandom number generator: the coordinates and directions of the velocities were set in accordance with a uniform distribution; the velocity modules were set in accordance with the Maxwell distribution for a given temperature *T*. In some calculations, the velocity modules were taken to be the same (see below for more details). For each particle, the Newton equation was written:

$$m_i \cdot \frac{d^2 \vec{r}_i(t)}{dt^2} = -G \cdot m_i \cdot \sum_{j \neq i}^{n} \frac{m_j \cdot (\vec{r}_i - \vec{r}_j)}{|\vec{r}_i - \vec{r}_j|^3}$$

where $m_i$ is the mass of the *i*-th particle $\vec{r}_i$ is the radius-vector of the *i*-th particle, and *G* is the gravitational constant.

The system of first-order ordinary differential equations was solved by the Runge-Kutta method of the 6th order (program code DVERK [126]) with an adaptive change in the size of the integration step. The choice of a single-step integration method is due to the absence of the need for a method initialization procedure, whereas multi-step methods require several preliminary steps, which can significantly affect the calculation speed when processing additional events.

In this simulation, it was important to keep the particles within a given volume (cube), which requires additional tracking and processing of events (attempts of particles to leave the volume). The retention of particles in the cube was carried out by the "mirror" reflection of the particle from the wall of the cube and was realized by changing the sign of the corresponding component of the particle velocity to the opposite when the particle reached the boundary of the set volume. In other words, if a particle went beyond the cube boundary, then the moment of its crossing the boundary was determined and from that moment on, simulation continued but with the opposite sign in the velocity component normal to the wall. Note that the periodic boundary conditions, which are often used in molecular-kinetic calculations and are more computationally convenient, are not suitable for studying the fundamental properties of the behavior of systems (see [127] for more details).

The interaction between the particles was calculated by a complete running over all the particles without any modifications (simplifications) in the nature of the interaction. The quality of integration of the differential equations was controlled by monitoring the total energy of the particle system. In the calculations, the change in the total energy of the particle system did not exceed 0.1% of the initial value.

### 2.2 Relaxation Processes

Systems with long-range interaction, which include the gravitational and Coulomb interactions, are usually described by the ideality parameter

$$\delta = \langle U \rangle / \langle W_k \rangle,$$

(or $\gamma \cong \delta^3$) representing the ratio of the average potential $\langle U \rangle$ energy of the system to its average kinetic energy $\langle W_k \rangle$ and characterizing the degree of the ideality of the system under consideration. In the ideal case, $\delta << 1$, the consideration of many issues related to the behavior of such systems is usually significantly simplified.

To consider the relaxation, a series of calculations was performed in non-ideal $\delta >> 1$ and almost ideal $\delta \approx 1$ cases.

*Relaxation time.* In works [128, 129] the relaxation time $\tau$ is considered in detail in an electron-ion plasma based on the analysis of the kinetic equations of the energy balance and is given by:

$$\tau = \frac{5}{4} \tau_{ei}$$

$$\tau_{ei} = \frac{3m_e^{1/2}T_e^{3/2}}{4\sqrt{2\pi}e^2e_i^2n_i\ln\Lambda}$$

where $m_e$ is the mass of the electrons, $T_e$ is the temperature of the electrons, $e$ is the charge of the electron, $e_i$ is the charge of the ion, and $\ln\Lambda$ is the Coulomb logarithm.

In the case of charges of the same mass, it is somewhat modified [128, 129].

$$\frac{1}{\tau} = \frac{4}{5}\frac{1}{\tau_{ei}}\left(1+\left|\frac{e}{e_i}\right|\frac{1}{\sqrt{2}}\right)$$

When moving to the case of the gravitational interaction of particles of the same mass, it is necessary to replace the factor $e_ie$, which corresponds to the potential of the Coulomb interaction of electric charges, with the factor $Gm^2$, which corresponds to the potential of the gravitational interaction of masses.

$$\tau = \frac{15T^{3/2}}{16\sqrt{\pi}m^{7/2}G^2n\ln\Lambda} \quad (4)$$

where $m$ is the mass of a particle, $G$ is the Newtonian gravitational constant, $n$ is the particle concentration, $T$ is the temperature, and $\ln\Lambda$ is the Coulomb logarithm. A thorough consideration of the issues related to the problem of cutting the Coulomb logarithm in plasma is presented in [130]. In the case of gravitational particles the Coulomb logarithm according [131, 132] is

$$\ln\Lambda = \ln\left(\frac{r_{\max}}{r_0}\right)$$

where $r_{max}$ is a certain upper limit of the impact parameter, $r_0$ is the impact parameter at which the deflection of the particle during scattering occurs at an angle of 90°. In [131], for the Coulomb logarithm, the expression

$$\ln\Lambda = \ln(0.4N) \quad (5)$$

is recommended, where $N$ is the total number of particles in the system. When applied to compact star formations with stars of approximately equal masses, the Coulomb logarithm is given by

$$\ln\Lambda = \ln\left(\frac{D_0v^2}{2Gm}\right)$$

where for spherical systems $D_0$ is comparable in order of magnitude to the radius of this system, for clusters in the form of a disk $D_0$ is comparable in order of magnitude to the thickness of the disk, $v^2$ is the typical square of the relative velocity [132]. Note that there is no unambiguously accepted expression for the relaxation time. In various monographs (see, for example, [131-134]), one can find other representations for the relaxation time, which differ mainly either by using different estimation methods or by accurately considering other statements of the relaxation problem, for example, in the framework of the Fokker-Planck equation [131]. The most voluminous derivation for the relaxation time (16 pages of the text) seems to belong to S. Chandrasekhar [134]. Of course, all these expressions differ from each other by a factor of the order of one. Because of this, we need to make a specific choice between them, so in this paper, we will use the expressions (4) and (5) for the relaxation time.

## 3. MODELING AND DISCUSSION OF THE RESULTS

*Case A* ($\delta \approx 1$). The calculation parameters were as follows: the number of particles was 300, the mass of a particle was $10^{-9}$ kg, the length of the half edge of the cube was $5 \times 10^{-7}$ m, the initial kinetic energy of all the particles was the same $W_k$ = 0.05 eV. The particle velocities were set to be the same at the initial moment of time so that the evolution (see Figs. 1, 2) of the distribution functions of the particles over the total and kinetic energy (or velocities) could be seen. Under the conditions listed above, the potential energy at the initial moment of time was $U$ = –35.7 eV (that corresponds to the average potential energy $\langle U\rangle$ = –0.119 eV), so that the modulus of the ideality parameter at the initial moment of time was 2.4. Substituting the value $T = \langle W_k\rangle/1.5$ for the temperature in (4) and (5) one can obtain the relaxation time of $\tau$ = 2.88 s. This value is in good agreement with the results of our numerical modeling. Indeed, the particle distribution was almost completely formed by 1-2 s. It had the form of Maxwellian distribution with a temperature of approximately 0.08 eV. At the initial moment of time, it is incorrect to talk about the temperature of the particles since all the particles had the same kinetic energy. It makes

sense to talk about the temperature only at the time $\tau$, for which the ideality parameter is already $\langle U\rangle/\langle W_k\rangle = 1.66$.

The relaxation time obtained in the numerical simulations coincides well with the time obtained in the theoretical consideration of relaxation processes. However, the question arises: until what time should the calculations be performed to be completely sure that the relaxation process is over? It may turn out that the system will continue to evolve. For example, the relaxation time in numerical experiments can be significantly longer than $\tau$, and then on time scales of the order of $\tau$, the changes in the distribution function are simply almost imperceptible. In this case, the virial theorem comes to the rescue. In systems with gravitational interaction, the average value of $\delta$ should be 2 if particles occupy a limited volume. If we remove the walls in the calculations, then $\delta$ goes to the value of 2 by about 0.5 s and then does not change. At the same time, the particles continue to move in a compact region without escaping, despite the absence of any boundaries. Consequently, there will be no further redistribution of the values of the kinetic and potential energies in the system.

Therefore, in Fig. 1d and 2d we have the final form of the distribution functions established in the system. As for Fig. 1d, then we see nothing surprising here – the kinetic energy distribution function has acquired a Maxwellian form, as it should be. As for the function in Fig. 2d, then it is in no way similar to the canonical Gibbs distribution (1). The reasons for this have been discussed in sufficient detail above, so we will not repeat them.

*Case B* ($\delta >> 1$). The calculation parameters were as follows: the number of particles was 300; the particle mass is $10^{-7}$ kg; the particle temperature was $T = 10^{-2}$ eV (initially the Maxwellian distribution); the length of the half edge of the cube is $5\ 10^{-3}$ m. In this case, the particle velocities were not set to be the same at the initial moment of time. As can be seen (Figs. 3, 4), there are no significant differences in the evolution of the particle distribution functions over the total and kinetic energies (or velocities) in comparison with the previous case. Under the conditions listed above, the potential energy at the initial time was $U = -36.2$ eV, so that the modulus of the ideality parameter at the initial time was 8.04 (if we take into account that $\langle W_k\rangle = 1.5T$), while the relaxation time according to (4) and (5) should be $1.34\ 10^4$ s. This value is in good agreement with the result of the numerical modeling. Indeed, the particle distribution was almost completely formed by $1.5\times10^4$-$2\times10^4$ s. It had the form of Maxwellian distribution with a temperature of approximately 0.12 eV.

The relaxation time obtained in the numerical simulations coincides well with the time obtained in the theoretical consideration of relaxation processes. Just as in the first case, the average value of $\delta$ should be 2. If we remove the walls in the calculations, then this ratio goes to the value of 2 to about 25000 s and then does not change. Therefore, there will be no further redistribution of the values of the kinetic and potential energies in the system. Just as in the first case, the particles continue to move in a limited area without escaping.

Here (Fig. 3d, 4d) the situation is almost exactly the same as in the case A (see the description of case A in more detail) for the distribution functions considered in it (Figs. 1d, 2d). The kinetic energy distribution function becomes Maxwellian (Fig. 3d), while the total energy distribution function is far from the canonical Gibbs distribution (Fig. 4d).

Earlier we noted (see section Systems in equilibrium) that if we carry out the analysis in the language of entropy, then in equilibrium in the case of the canonical Gibbs distribution, the system should have the largest possible entropy, i.e. to be in the greatest degree of disorder. It is believed that only in this case, when the system has a Gibbs distribution, we can assert that the system is in equilibrium. The total energy is reduced into terms that depend on velocities (kinetic energy) and coordinates (potential energy). This, in turn, leads to the fact that in this state the velocity distribution in the system should be Maxwellian. From the present simulation, we see that the system has come to equilibrium, its distribution function is not canonical Gibbs distribution, but, nevertheless, the velocity (or kinetic energy) distribution is Maxwellian. In other words, the greatest degree of disorder in the system occurs not over the entire energy, but only over its kinetic component.

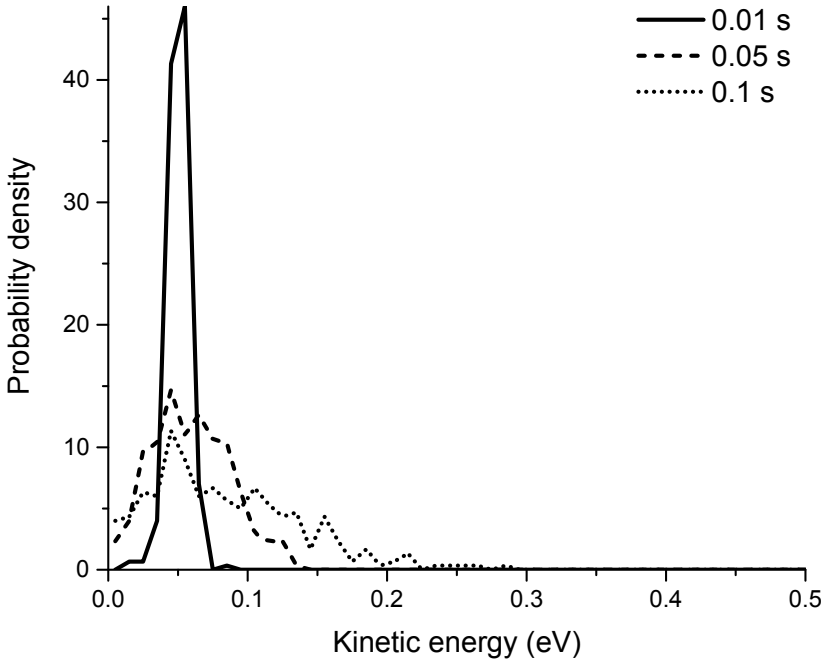


**Fig. 1a. Case A. Distribution functions of particles on the kinetic energy at times 0.01 (solid), 0.05 (dashed), and 0.1 s (dotted).**

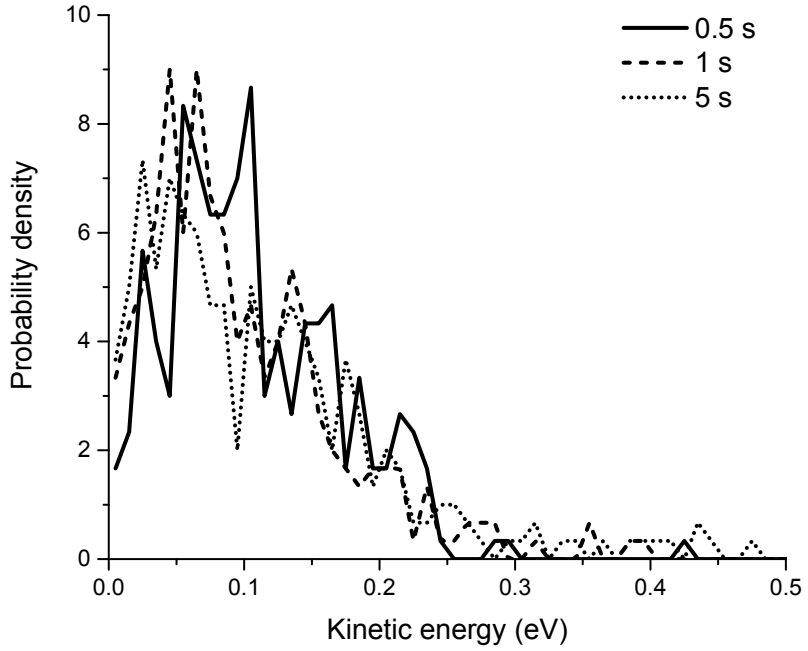


**Fig. 1b. Case A. Distribution functions of particles on the kinetic energy at times 0.5 (solid), 1 (dashed), and 5 s (dotted).**

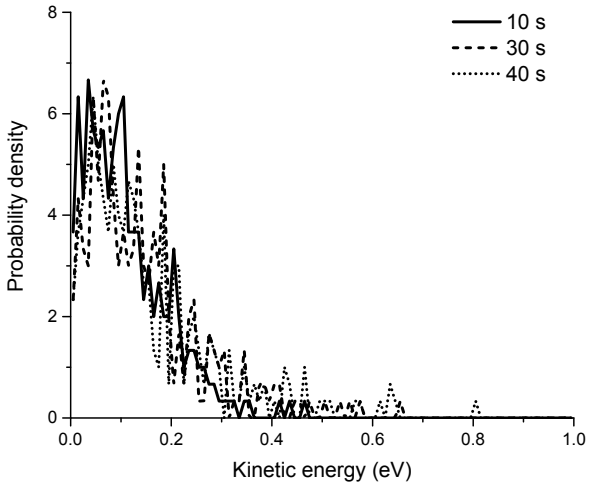


**Fig. 1c. Case A. Distribution functions of particles on the kinetic energy at times 10 (solid), 30 (dashed), and 40 s (dotted).**

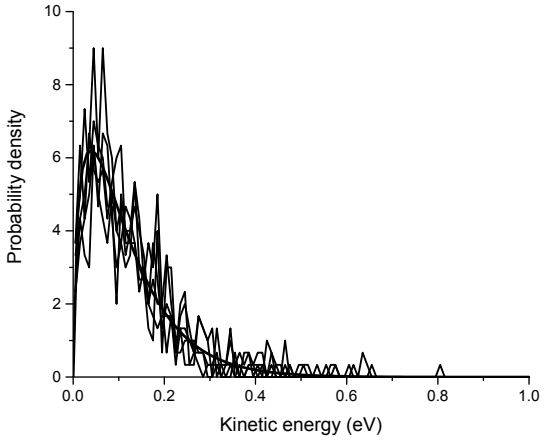


**Fig. 1d. Case A. Distribution functions of particles on the kinetic energy at times 1-40 s (thin lines), Maxwell distribution function for temperature of 0.08 eV (heavy line). See Figs. 1b, 1c for details.**

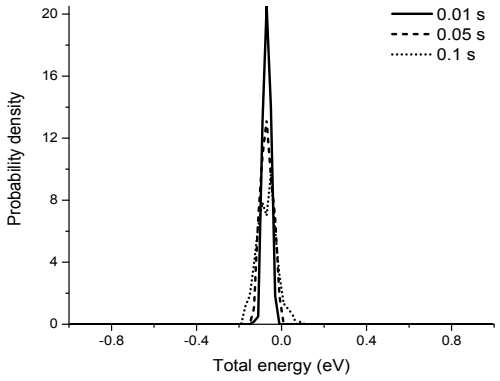


**Fig. 2a. Case A. Distribution functions of particles on total energy at times 0.01 (solid), 0.05 (dashed), and 0.1 s (dotted)**

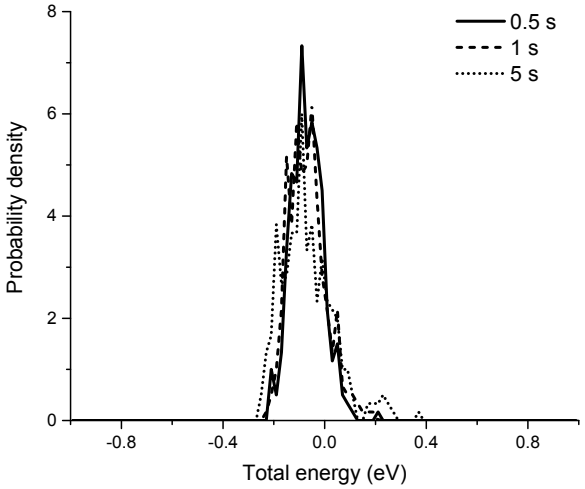


**Fig. 2b. Case A. Distribution functions of particles on total energy at times 0.5 (solid), 1 (dashed), and 5 s (dotted).**

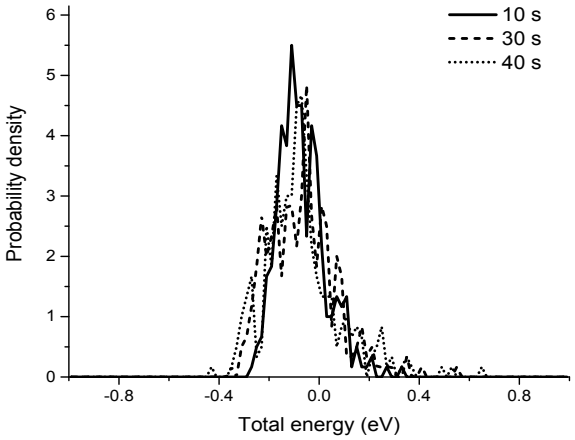


**Fig. 2c. Case A. Distribution functions of particles on total energy at times 10 (solid), 30 (dashed), and 40 s (dotted)**

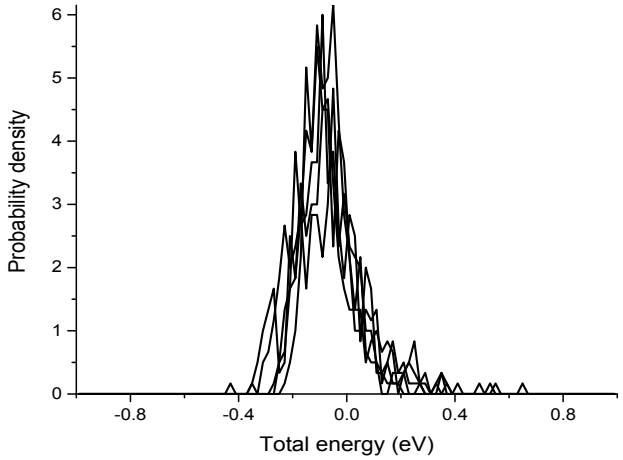


**Fig. 2d. Case A. Distribution functions of particles on total energy at times 1-40 s. See Figs. 2b, 2c for details**

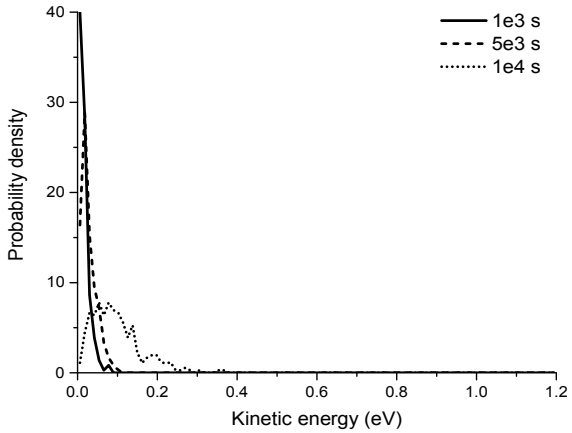


**Fig. 3a. Case B. Distribution functions of particles on the kinetic energy at times 1000 (solid), 5000 (dashed), and $1 \cdot 10^4$ s (dotted)**

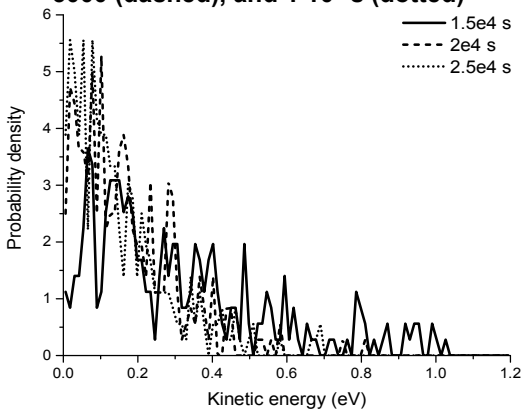


**Fig. 3b. Case B. Distribution functions of particles on the kinetic energy at times $1.5 \cdot 10^4$ (solid), $2 \cdot 10^4$ (dashed), and $2.5 \cdot 10^4$ s (dotted)**

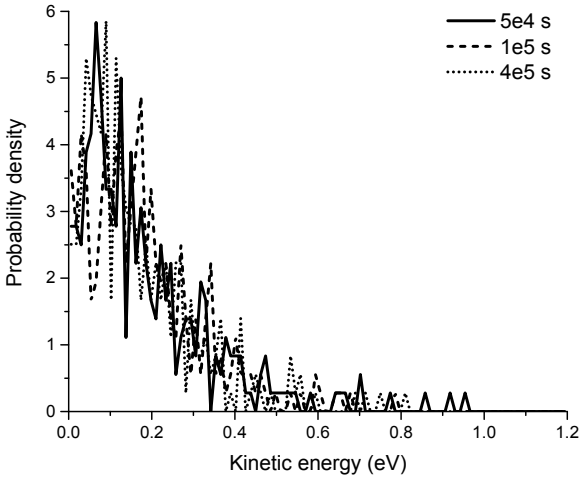


**Fig. 3c. Case B. Distribution functions of particles on the kinetic energy at times $5 \cdot 10^4$ (solid), $1 \cdot 10^5$ (dashed), and $4 \cdot 10^5$ s (dotted)**

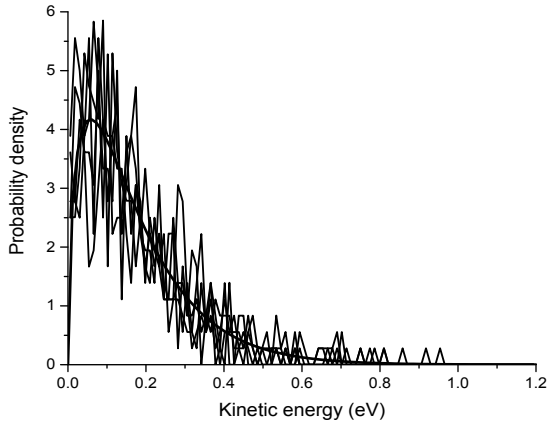


**Fig. 3d. Case B. Distribution functions of particles on the kinetic energy at times $2 \cdot 10^4$-$4 \cdot 10^5$ s (thin lines), Maxwell distribution function for temperature of 0.12 eV (heavy line). See Figs. 3b, 3c for details**

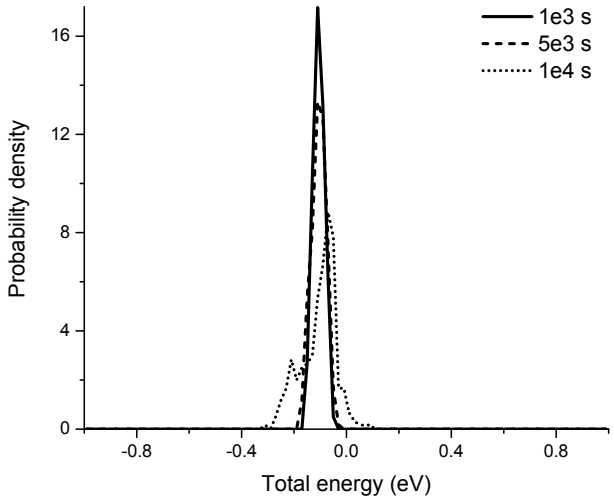


**Fig. 4a. Case B. Distribution functions of particles on total energy at times 1000 (solid), 5000 (dashed), and $1 \cdot 10^4$ s (dotted)**

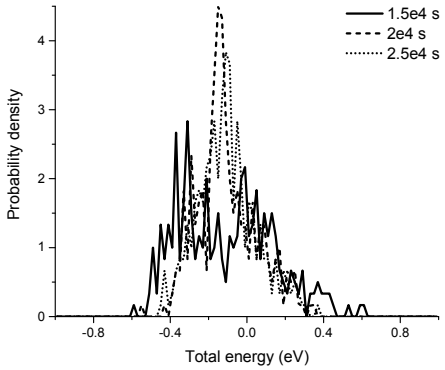


**Fig. 4b. Case B. Distribution functions of particles on total energy at times $1.5 \cdot 10^4$ (solid), $2 \cdot 10^4$ (dashed), and $2.5 \cdot 10^4$ s (dotted)**

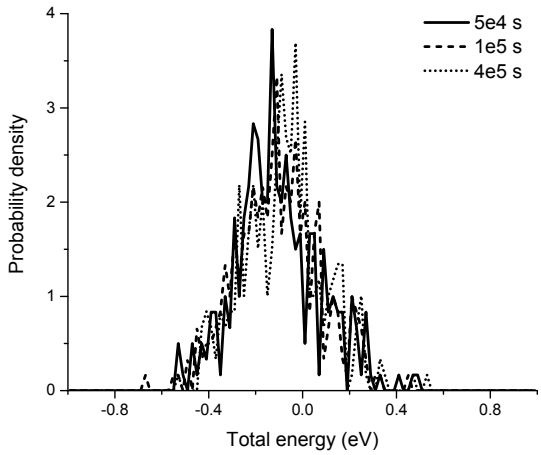


**Fig. 4c. Case B. Distribution functions of particles on total energy at times $5 \cdot 10^4$ (solid), $1 \cdot 10^5$ (dashed), and $4 \cdot 10^5$ s (dotted)**

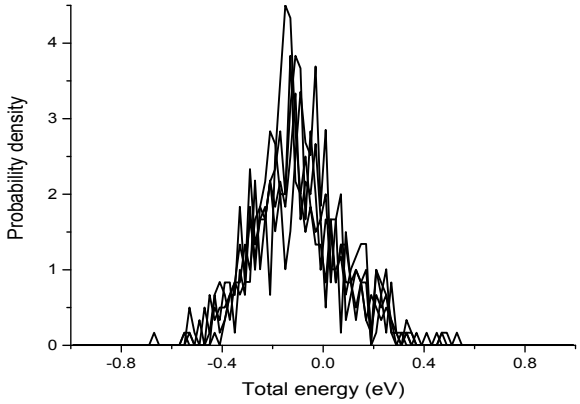


**Fig. 4d. Case B. Distribution functions of particles on total energy at times $2 \cdot 10^4$-$4 \cdot 10^5$ s. See Figs. 4b, 4c for details**

## 4. CONCLUSIONS

Numerical simulation of the behavior of classical particles interacting gravitationally with each other was carried out. The conducted modeling is directly related to the question of the possibility of substantiating the statistical behavior of classical mechanical systems on the basis of their mechanical behavior. The cases with the ratio of the potential energy to the kinetic energy of $\delta \geq 1$ at the initial moment of time were considered. The main results obtained in the work can be summarized as follows.

1. The relaxation time of the system obtained on the basis of the numerical simulation is in good agreement with the relaxation time obtained in a large number of studies using various approximations.
2. However, purely the gravitational interaction of particles does not lead to the formation of the particle energy distribution function of Boltzmann form in the region of large magnitude negative energies. Such a behavior must occur in accordance with the known theorems when the system approaches its equilibrium. Almost more than a century of attempts have been made to justify such a transition and to concordance the statistical and mechanical approaches in such a transition.
3. On the other hand, such a behavior should not also occur in accordance with known theorems. The transition to the equilibrium corresponds, in the language of entropy, to the transition of the system to the state with the highest possible entropy. However, the well-known entropy conservation theorems

of mechanical closed systems in both the classical and quantum cases prevent such a transition since the entropy of such systems cannot change.

4. Thus, the current work demonstrates another example of a system in which, when approaching its equilibrium, the distribution function does not take the canonical form if the system does not involve any other factors besides its mechanical behavior. Earlier, a similar result was demonstrated by the example of a classical Coulomb plasma.
5. Thus, it is generally impossible to justify the transition of the distribution function of a closed system in equilibrium to the canonical Gibbs distribution only on the basis of its mechanical behavior for systems with long-range Coulomb or gravitational interactions. For such a justification, it is necessary to take into account the presence of other processes in the system in addition to only the mechanical interaction of particles with each other. For example, the system must have a stochastiser in one form or another, which will remove the ban on changing the entropy of closed systems imposed by the well-known entropy conservation theorems.

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## COMPETING INTERESTS

Authors have declared that no competing interests exist.

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