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Research Article

Gravitational Collapse and Singularity Removal in Rastall Theory

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In the present work, we study spherically symmetric gravitational collapse of a homogeneous fluid in the framework of Rastall gravity. Considering a nonlinear equation of state (EoS) for the fluid profiles, we search for a class of nonsingular collapse solutions and the possibility of singularity removal. We find that depending on the model parameters, the collapse scenario halts at a minimum value of the scale factor at which a bounce occurs. The collapse process then enters an expanding phase in the postbounce regime, and consequently the formation of a spacetime singularity is prevented. We also find that, in comparison to the singular case where the apparent horizon forms to cover the singularity, the formation of apparent horizon can be delayed allowing thus the bounce to be causally connected to the external universe. The nonsingular solutions we obtain satisfy the weak energy condition (WEC) which is crucial for physical validity of the model.

1. Introduction

The process of gravitational collapse of a massive object and its final outcome is one of the central questions in relativistic astrophysics and gravitation theory. In the framework of general relativity (GR), the Hawking and Penrose singularity theorems predict that under physically reasonable conditions, a continual collapse process leads to the formation of a spacetime singularity, that is, a spacetime event where densities and spacetime curvatures grow limitlessly and diverge [1]. During the last years, much attempts have been directed towards exploring different aspects of the gravitational collapse process and the studies along this line of research indicate that the spacetime singularity that forms as the collapse end product could be dressed by a spacetime event horizon (black hole formation) or visible by the observers in the universe (naked singularity formation) [2]. Usually, formation of a naked singularity as the collapse outcome is considered the violation of the cosmic censorship conjecture [3–5]. This conjecture states that singularities that form as the collapse final state will always be hidden by the event horizon of a black hole and cannot be visible to the observers in the universe [6] (see [7] for a recent review on this conjecture). However, during the past decades, many examples of naked singularity formation as possible counterexamples to the cosmic censorship conjecture have been reported in the literature, among which we can quote gravitational collapse of dust, perfect fluids, and radiation shells [8, 9]. Such a study has been extended to gravitational collapse in the presence of a cosmological constant term [10], higher dimensional collapse models [11–14], higher-order gravity theories [15], scalar field collapse [16–20], and self-similar collapse [21–23] (see also [2] for a recent review). Also, in the context of modified gravity theories, it is shown that naked singularities could form depending on different aspects of the theory (see, e.g., [24–30]).

Despite the fact that the formation of naked singularities may provide us a useful observational testbed for detecting high energy phenomena, these objects seem unpleasant as classical GR breaks down at the spacetime singularity. However, it is generally believed that such a singularity that forms in a classical regime can be avoided once quantum gravity corrections are taken into account. In this regard, a great amount of work has been devoted to investigate nonsingular collapse models, for example, corrections that may arise in the strong field regime, as obtained in the framework of some Loop Quantum Gravity (LQG) models [31–36]. Work along this line has been also extended to modified gravity models, e.g., singularity avoidance in Eddington-inspired

Born-Infeld theory [37], modified Gauss-Bonnet gravity [38, 39], Horava-Lifshitz gravity [40], nonminimal coupling of classical gravity with fermions [41], and other modified gravity theories [42].

In the light of the above considerations, one may be motivated to study modified gravity theories in the context of which the collapse scenario leads to a nontrivial outcome, different to singular collapse settings that have been studied in GR [9]. In this regard, one can generalize the standard GR to include a nonminimal coupling between geometry and matter fields. As we know, in most of the modified gravity theories, the energy-momentum source is characterized by a divergence-free tensor field which couples to the geometry in a minimal way [43, 44]. However, such a property of the energy-momentum tensor (EMT) which leads to the energy-momentum conservation law is not obeyed by the particle production process [45–48]. Hence, it seems reasonable to assume a nonvanishing divergence for EMT and seek for a modified gravitational theory whose geometrical degrees of freedom (not present in GR) may affect the final fate of the collapse. In this regard, one may relax the condition on EMT conservation law; i.e., mathematically the relation $\nabla_\mu T^\mu{}_\nu = 0$ is not valid anymore [49–55]. This idea was firstly put forward by Peter Rastall [54] who proposed a gravitational model in which the divergence of $T^\mu{}_\nu$ is proportional to the gradient of the Ricci scalar, i.e., $\nabla_\mu T^\mu{}_\nu \propto \nabla_\nu R$, so that the usual conservation law is recovered in the flat spacetime. This kind of modified gravity model has attracted a great deal of attention recently and is in good agreement with various observational data and theoretical expectations [56]. In the present article, we are motivated to investigate a simple model for gravitational collapse of an isotropic and homogeneous matter distribution with nonlinear EoS in Rastall gravity. We therefore proceed with considering the field equations in Rastall gravity in Section 2 and search for nonsingular collapse solutions in Section 3. Our conclusions are drawn in Section 4.

2. Field Equations of Rastall Gravity

According to the original idea of Rastall [54], the vanishing of covariant divergence of the matter energy-momentum tensor is no longer valid and this vector field is proportional to the covariant derivative of the Ricci curvature scalar as

$$\nabla_a T^a_b = \lambda \nabla_b \mathscr{R}, \tag{1}$$

where λ is the Rastall parameter. The Rastall field equations are then given by [54, 57]

$$G_{ab} + \gamma g_{ab} \mathscr{R} = \kappa T_{ab}, \tag{2}$$

where $\gamma = \kappa\lambda$ is the Rastall dimensionless parameter and κ is the Rastall gravitational coupling constant. The above equation can be rewritten in an equivalent form as

$$G_{ab} = \kappa T^{\mathrm{eff}}_{ab}, \tag{3}$$

where

$$T^{\mathrm{eff}}_{ab} = T_{ab} - \frac{\gamma T}{4\gamma - 1} g_{ab} \tag{4}$$

is the effective energy-momentum tensor whose components are given by [58, 59]

$$T^{\mathrm{eff}0}_{0} \equiv -\rho^{\mathrm{eff}} = -\frac{(3\gamma - 1)\rho + \gamma(p_r + 2p_t)}{4\gamma - 1}, \tag{5}$$

$$T^{\mathrm{eff}1}_{1} \equiv p^{\mathrm{eff}}_r = \frac{(3\gamma - 1)p_r + \gamma(\rho - 2p_t)}{4\gamma - 1}, \tag{6}$$

$$T^{\mathrm{eff}2}_{2} = T^{\mathrm{eff}3}_{3} \equiv p^{\mathrm{eff}}_t = \frac{(2\gamma - 1)p_t + \gamma(\rho - p_r)}{4\gamma - 1}. \tag{7}$$

It is noteworthy that in the limit of $\lambda \to 0$, the standard GR is recovered. Moreover, for an electromagnetic field source, we get $T^{\mathrm{eff}}_{ab} = T_{ab}$ leading to $G_{ab} = \kappa T_{ab}$. Therefore, the GR solutions for $T = 0$, or equivalently $R = 0$, are also valid (the Rastall gravity) [57, 60].

3. Solutions to the Field Equations

In the framework of classical GR, the continual gravitational collapse of a massive body under its own weight was investigated for the first time by Oppenheimer, Snyder, and Datt (OSD) [61, 62]. They considered the evolution of a spherically symmetric homogeneous dust cloud which starts from rest. The interior spacetime of such a collapse setting can be described by the Friedman-Robertson-Walker metric given by

$$ds^2 = -dt^2 + \frac{a^2(t)}{1 - kr^2} dr^2 + R^2(t, r) d\Omega^2, \tag{8}$$

where k determines the spatial curvature, $R(t, r) = ra(t)$ is the physical radius of the collapsing object, with $a(t)$ being the scale factor, and $d\Omega^2$ is the standard line element on the unit 2-sphere. The EMT of a pressureless matter is simply given by $T^a_b = \mathrm{diag}\,(-\rho, 0, 0, 0)$, from which one can find the Einstein field equations as

$$\frac{3k}{a^2} + 3\frac{\dot{a}^2}{a^2} = 2\kappa_G \rho, \tag{9}$$

$$\frac{k}{a^2} + \frac{\dot{a}^2}{a^2} + 2\frac{\ddot{a}}{a} = 0, \tag{10}$$

where $\kappa_G = 4\pi G$. Also, the conservation of EMT ($\nabla_\alpha T^\alpha_\beta = 0$) gives $\rho = C/a^3$, where C is a constant. Substituting them for energy density into equation (9) along with defining the conformal time $d\eta = dt/a$, we arrive at the following solution for the scale factor:

$$a(\eta) = \frac{a_i}{2}(1 + \cos(\eta)), t(\eta) = \frac{a_i}{2}(\eta + \sin(\eta)), \tag{11}$$

where $0 \leq \eta \leq \pi$. The above solution describes the collapse process of a homogeneous dust fluid for which the scale factor starts from the finite value a_i at $(\tau, \eta) = (0, 0)$ and becomes zero at $(\tau, \eta) = (\pi a_i/2, \pi)$. The vanishing of the scale factor at a finite time signals the formation of a spacetime singularity, i.e., a spacetime event at which the energy density and curvature get arbitrary large values and diverge. It can be shown that the singularity in the OSD model is necessarily hidden by an event horizon and thus a black hole is formed as the end state of a homogeneous dust collapse (see, e.g., [63] for a review on the OSD model).

In the present section, we seek for a class of homogeneous collapse solutions for which the formation of spacetime singularity is avoided. We shall see that this is possible in case we generalize the OSD model from GR to Rastall gravity along with assuming a nonlinear EoS for the fluid pressure. To this aim, we begin with a homogeneous and isotropic interior line element representing a spatially nonflat FLRW geometry. The field equations for an isotropic source ($T^a_b = \mathrm{diag}\,(-\rho, p, p, p)$) then read

$$3\frac{\dot{a}^2}{a^2} + \frac{3\mathrm{k}}{a^2} = \kappa\rho_{\mathrm{eff}} = \frac{2\kappa_G}{6\gamma - 1}[(3\gamma - 1)\rho + 3\gamma p], \tag{12}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = -\kappa p_{\mathrm{eff}} = \frac{2\kappa_G}{6\gamma - 1}[(1 - \gamma)p - \gamma\rho]. \tag{13}$$

Applying the Bianchi identity on equation (3) leaves us with the following continuity equation in Rastall gravity as

$$\left(\frac{3\gamma - 1}{4\gamma - 1}\right)\dot{\rho} + \left(\frac{3\gamma}{4\gamma - 1}\right)\dot{p} + 3H(\rho + p) = 0. \tag{14}$$

Next, we proceed to build and study collapse scenarios assuming a polytropic EoS $p = \alpha\rho^\beta$, where α and β are constants. Equation (14) can be solved for this EoS, and the solution reads

$$\ln\,(a) + \ln\left(\rho^{\beta(a_1 + a_2)/3(\beta - 1)}\left(\rho + \alpha\rho^\beta\right)^{-(a_1 + \beta a_2)/3(\beta - 1)}\right) + C_0 = 0, \tag{15}$$

where C_0 is an integration constant and

$$a_1 = \frac{1 - 3\gamma}{1 - 4\gamma}, a_2 = \frac{3\gamma}{1 - 4\gamma}, \beta \neq 1. \tag{16}$$

In order to find an explicit expression for energy density, we set $a_1 = -\beta a_2$ within equation (15). This gives

$$\rho(a) = \rho_i\left(\frac{a}{a_i}\right)^{3(4\gamma - 1)/(3\gamma - 1)}, \tag{17}$$

where $\rho_i = \rho(t_i)$ and $a_i = a(t_i)$ are the initial values of energy density and scale factor, respectively, and $t = t_i$ is the initial time at which the collapse begins. Equations (12) and (13) can then be rewritten as (we set the units so that $2\kappa_G = 1$)

$$3\frac{\dot{a}^2}{a^2} + \frac{3k}{a^2} = \frac{6\alpha\gamma\rho_i^{(3\gamma - 1)/3\gamma}}{6\gamma - 1}\left(\frac{a_i}{a}\right)^{(4\gamma - 1)/\gamma} + \rho_i\left(\frac{a_i}{a}\right)^{3(4\gamma - 1)/(3\gamma - 1)}, \tag{18}$$

$$\begin{aligned} 2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{2(1 - \gamma)}{6\gamma - 1}\rho_i^{(3\gamma - 1)/3\gamma}\left(\frac{a_i}{a}\right)^{(4\gamma - 1)/\gamma} \\ - \frac{2\gamma}{6\gamma - 1}\rho_i\left(\frac{a_i}{a}\right)^{3(4\gamma - 1)/(3\gamma - 1)}. \end{aligned} \tag{19}$$

Next, we proceed to study the collapse dynamics using numerical methods in order to solve equation (19). We further employ equation (18) to find the initial condition on the speed of collapse which is given by

$$\dot{a}(t_i) = -\left[\frac{3ka_i^2(1 - 6\gamma) + 2\rho_i a_i^4\left(3\gamma\left(1 + \alpha\rho_i^{-1/3\gamma}\right) - 1\right)}{3a_i^2(6\gamma - 1)}\right]^{1/2}. \tag{20}$$

Figure 1(a) shows the numerical solution to equation (19) for a closed geometry ($k = 1$) and different values of the Rastall parameter. As we observe, the collapse starts its evolution by a finite velocity, i.e., $\dot{a}_i < 0$ and continues through a contracting regime until the bounce time $t = t_b$ is reached. At this time, the collapse process halts at a nonzero minimum value of the scale factor so that we have $a_{\mathrm{min}} = a(t_b)$ and $\dot{a}(t_b) = 0$ (see also Figure 1(b)). This minimum value of the scale factor can be obtained through equation (18) as

$$\frac{2\rho_i}{6\gamma - 1}\left(\frac{a_i}{a_{\mathrm{min}}}\right)^4\left[\left(\frac{a_i}{a_{\mathrm{min}}}\right)^{1/(3\gamma - 1)}(3\gamma - 1) + 3\left(\frac{a_i}{a_{\mathrm{min}}}\right)^{-1/\gamma}\alpha\gamma\rho_i^{-1/3\gamma}\right] - 3a_{\mathrm{min}}^{-2} = 0. \tag{21}$$

From Figure 1(a), we also note that the Rastall parameter could change the minimum value of the scale factor and the bounce time in such a way that the larger the values of the γ parameter, the greater the value of a_{min} and the sooner the bounce occurs. For $t > t_b$, the contracting regime switches to an expanding regime and the collapsing body disperses as the time passes. We also observe that the case of $\gamma = 0$ corresponds to the GR limit of the theory where the gravitational collapse process leads to singularity formation (see the dashed curve). Figure 1(c) shows the behavior of collapse acceleration. We therefore observe that the collapse experiences four phases during its dynamical evolution. (i) During the time interval at which $\dot{a} < 0$ and $\ddot{a} < 0$, the collapse undergoes an accelerated contracting regime. (ii) As time goes by, the collapse enters a decelerated contracting regime where $\dot{a} < 0$ and $\ddot{a} > 0$. (iii) After the bounce time, the collapse turns into an accelerated expanding phase for which $\dot{a} > 0$ and $\ddot{a} > 0$. (iv) Finally, at later times, the collapse enters a decelerated expanding regime where $\dot{a} > 0$ and $\ddot{a} < 0$.

For the sake of physical reasonability, we require that the weak energy condition (WEC) be satisfied. According to this condition, the energy density as measured by any local observer must be positive. Hence, for the energy-

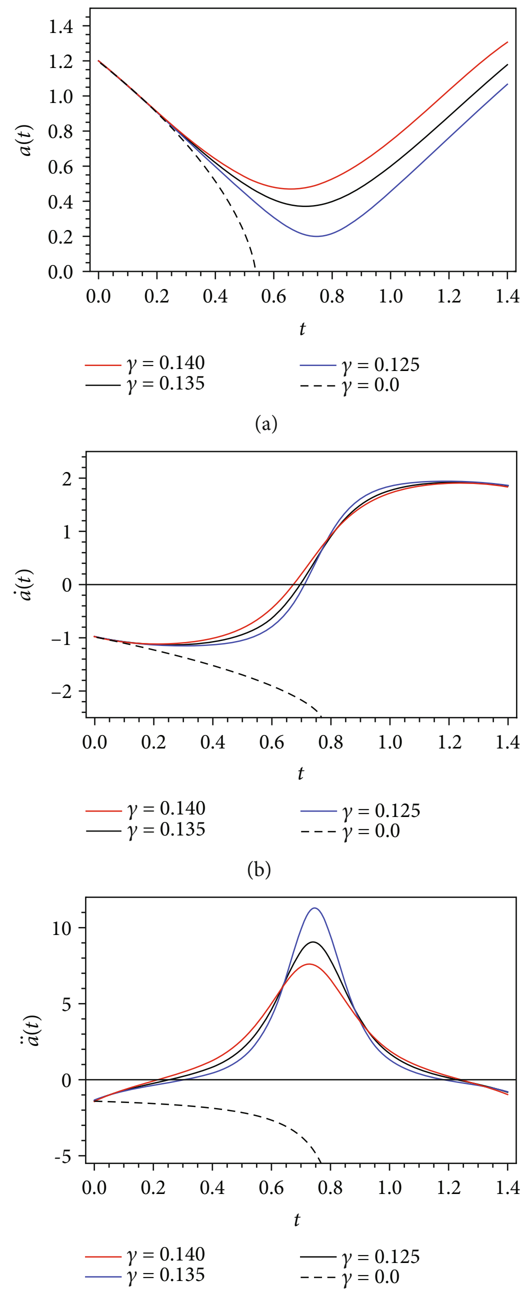

Figure 1: (a) The behavior of the scale factor over time for $a_i = 1.2$, $\rho_i = 2.0$, and $\alpha = 4.24$. (b) The behavior of speed of collapse for the same values of the parameters as chosen above. (c) The behavior of acceleration of collapse for the same values of the parameters as chosen above.

momentum tensor of ordinary matter and the effective fluid, the conditions

$$\rho \geq 0, \rho + p \geq 0, \tag{22}$$

$$\rho_{\text{eff}} \geq 0, \rho_{\text{eff}} + p_{\text{eff}} \geq 0 \tag{23}$$

must be satisfied along any nonspacelike vector field. In Figure 2(a), we have plotted for energy density the WEC

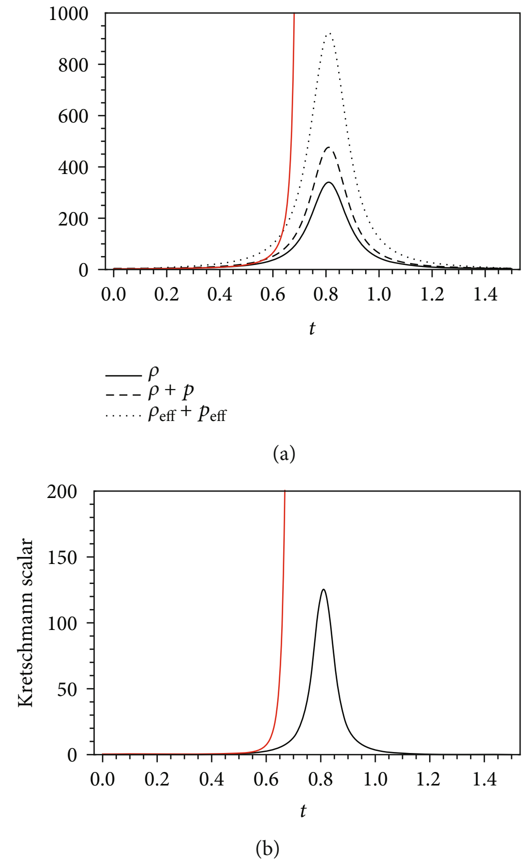

Figure 2: (a) The behavior of energy density over time for $a_i = 1.2$, $\rho_i = 2.0$, $\alpha = 4.24$, and $\gamma = 0.14$. The red curve stands for the case with $\gamma = 0$. (b) The behavior of the Kretschmann scalar for the same values of the parameters as chosen above.

for ordinary EMT and the WEC for effective EMT. We also observe that in the limit where $\gamma \to 0$, the energy density diverges signaling the occurrence of a spacetime singularity (see the red curve). Another quantity that the divergence of which implies singularity formation is the Kretschmann scalar defined as

$$\begin{aligned} \mathscr{K} &= \mathscr{R}_{\alpha\beta\delta\varepsilon}\mathscr{R}^{\alpha\beta\delta\varepsilon} = \dot{H}^2 + 2H^4 + 2H^2\dot{H} \\ &= \frac{4\ell(1-3\gamma(+w))^2}{81(1+w)^4(1-4\gamma)^4(t-t_s)^4}. \end{aligned} \tag{24}$$

In Figure 2(a), we have plotted for the behavior of this quantity where we observe that the Kretschmann scalar behaves regularly and is finite throughout the collapse process (black solid curve), while, for $\gamma = 0$ (red curve), this quantity grows unboundedly and diverges at the singularity. We also note that for $\beta = 0$ the results of [64] will be recovered.

An important issue that needs to be examined in each collapse setting is the study of dynamics of apparent horizon and causal structure of spacetime during the evolution of the collapse process. The apparent horizon is the outermost boundary of the trapped region, and the condition for its

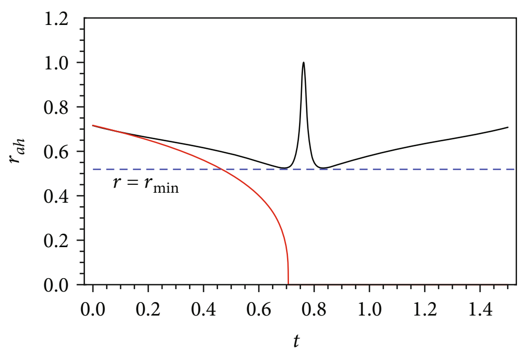

FIGURE 3: The behavior of the apparent horizon curve over time for $a_i = 1.2$, $\rho_i = 2.0$, $\alpha = 4.24$, and $\gamma = 0.14$. The red curve stands for the case with $\gamma = 0$.

formation is provided by the requirement that the surface with $R(t, r) =$ Constant is lightlike or in other words $g^{\mu\nu}\partial_\mu R \partial_\nu R = 0$ [9]. This condition for our model reduces to $\dot{R}^2 + r^2 = 1$ from which we can find the radius of the apparent horizon as

$$r_{ah} = \frac{1}{\sqrt{\dot{a}^2 + 1}}. \tag{25}$$

In Figure 3, we have plotted for the radius of the apparent horizon and compared the cases with $\gamma = 0.14$ and $\gamma = 0$. In the former (black solid curve), we observe that during the collapse process, the apparent horizon radius decreases to the minimum value $r = r_{\min}$ and then reaches a maximum value at the bounce time. It then converges to the same minimum radius in the postbounce regime. The apparent horizon curve will never vanish; i.e., it will never hit the singularity at $R = 0$, in contrast to the case with $\gamma = 0$ where the apparent horizon covers the singularity at a finite amount of time, leading to black hole formation (see the red curve).

4. Concluding Remarks

In this work, we studied the process of gravitational collapse of an isotropic homogeneous fluid which obeys a nonlinear EoS, i.e., $p = \alpha\rho^\beta$, between energy density and pressure profile. We found that, depending on the model parameters, nonsingular collapse solutions can be obtained in such a way that the collapse starts from regular initial data, proceeds for a while, and halts at a bounce time at which the scale factor reaches its minimum value. Then, after the bounce time is passed, the collapse scenario turns into an expanding phase. We further observed that the energy density and Kretschmann scalar behave regularly and are finite throughout the contracting and expanding regimes. In this regard, the spacetime singularity which is present in the OSD collapse model is avoided. Also, for the singular model, the apparent horizon necessarily forms to cover the singularity whereas in the model described herein, the initial radius of the collapsing matter $(R(t_i, r) = r)$ can be chosen as $r < r_{\min}$. In this case, the horizon formation is prevented, and thus, the bounce can be visible to faraway observers in the universe. As the Rastall parameter is a measure of ability of matter and curvature to interact with each other, we therefore conclude that such an ability can provide a setting in which the formation of spacetime singularities is avoided in a gravitational collapse process.

As the final remarks, it is noteworthy that the present collapse scenario can be compared to other collapse settings such as gravitational collapse of a homogeneous Weyssenhoff fluid in the framework of Einstein-Cartan gravity [65]. The Weyssenhoff fluid is a generalization of a perfect fluid in GR to include the intrinsic angular momentum (spin) of the fermionic matter field. Comparing equations (18) and (19) with the corresponding equations given in [65], we observe that for $\gamma = -1/2$, the collapse dynamics presented in this work mimics that of a Weyssenhoff fluid with EoS $w = 1/5$. Although a more detailed and in-depth analysis is needed in order to understand the correspondence between the two theories, one may intuitively imagine a possible relation between matter-curvature coupling in Rastall theory and spacetime torsion in Einstein-Cartan gravity. It is also worth mentioning that the exterior spacetime of the collapsing body can be obtained by matching the interior spacetime through a timelike hypersurface to an exterior Vaidya spacetime [66–68], using Israel-Darmois junction conditions [69]. By doing so, one can show that in the framework of the present study, the exterior region of the collapsing object is a Schwarzschild spacetime with dynamical boundary [64, 65].

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The author declares that he has no conflicts of interest.

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