mathematics

Article

New Results on F-Contractions in Complete Metric Spaces

Jelena Vujaković [1,*], Nataša Kontrec [1], Marina Tošić [1], Nicola Fabiano [2] and Stojan Radenović [3]

1 Faculty of Sciences and Mathematics, University of Priština in Kosovska Mitrovica, Lole Ribara 29, 38220 Kosovska Mitrovica, Serbia; natasa.kontrec@pr.ac.rs (N.K.); marina.tosic@pr.ac.rs (M.T.)
2 "Vinča" Institute of Nuclear Sciences, National Institute of the Republic of Serbia, University of Belgrade, Mike Petrovića Alasa 12-14, 11351 Belgrade, Serbia; nicola.fabiano@gmail.com
3 Faculty of Mechanical Engineering, University of Belgrade, Kraljice Marije 16, 11120 Beograd, Serbia; radens@beotel.net
* Correspondence: jelena.vujakovic@pr.ac.rs

Abstract: The main purpose of this paper is to improve, generalize, unify, extend and enrich the recent results established by Dung and Hang (2015), Piri and Kumam (2014, 2016), and Singh et al. (2018). In our proofs, we only use the property (F1) of Wardowski's F-contraction, while the many authors in their papers still use all tree properties of F-contraction as well as two new properties introduced by Piri and Kumam. Our approach in this paper indicates that for most results with F-contraction, property (F1) is sufficient. It is interesting to investigate whether (F1) is sufficient in the case of multi-valued mappings.

Keywords: F-contraction; generalized F-contraction; convex contraction; α-admissible mapping; triangular α-admissible mapping; fixed point; Ćirić's quasi-contraction

MSC: 47H10; 54H25; 54E50

check for updates

Citation: Vujaković, J.; Kontrec, N.; Tošić, M.; Fabiano, N.; Radenović, S. New Results on F-Contractions in Complete Metric Spaces. *Mathematics* **2022**, *10*, 12. https://doi.org/10.3390/math10010012

Academic Editor: Christopher Goodrich

Received: 19 November 2021
Accepted: 16 December 2021
Published: 21 December 2021

1. Introduction and Preliminaries

Fixed-point theory (metrical and topological) is one of the most important theories in nonlinear mathematical analysis. It began to develop in the late nineteenth century. It is based on concepts such as iteration, the Picard sequence, fixed points, common fixed points, continuous mapping on a bounded and closed subset of $\mathbb{R}^n$, et cetera. The first explicit result in metrical fixed-point theory appeared in 1922 by the Polish mathematician S. Banach. It was used to solve one type of integral equation. It is known in literature as the Banach contraction principle (BCP) [1] and has become a very popular and a fundamental tool in solving existing problems, arising not only in pure and applied mathematics but also in many branches of sciences, engineering, social sciences, economics and medical sciences. Furthermore, 1974's Ćirić's quasi-contraction [2] is one of the most frequently found generalizations of the Banach contraction principle. He considered all possible six values $\widetilde{d}(\widetilde{x},\widetilde{y}),\widetilde{d}(\widetilde{x},t\widetilde{x}),\widetilde{d}(\widetilde{y},t\widetilde{y}),\widetilde{d}(\widetilde{x},t\widetilde{y}),\widetilde{d}(\widetilde{y},t\widetilde{x})$ and $\widetilde{d}(t\widetilde{x},t\widetilde{y})$ by combining $\widetilde{x},\widetilde{y},t\widetilde{x},t\widetilde{y}$ for all $\widetilde{x},\widetilde{y}\in\mathcal{X}$ where t is self-mapping on a metric space $\left(\mathcal{X},\widetilde{d}\right)$. Namely, in [2], Ćirić formulated and proved the following result:

Theorem 1 ([2])**.** *Let $t:\mathcal{X}\to\mathcal{X}$ be a self-mapping on a complete metric space $\left(\mathcal{X},\widetilde{d}\right)$ and $k\in[0,1)$ such that for all $\widetilde{x},\widetilde{y}\in\mathcal{X}$*

$$\widetilde{d}(t\widetilde{x},t\widetilde{y})\leq k\cdot\max\left\{\widetilde{d}(\widetilde{x},\widetilde{y}),\widetilde{d}(\widetilde{x},t\widetilde{x}),\widetilde{d}(\widetilde{y},t\widetilde{y}),\widetilde{d}(\widetilde{x},t\widetilde{y}),\widetilde{d}(\widetilde{y},t\widetilde{x})\right\}. \quad (1)$$

Then t has a unique fixed point $\widetilde{x}^*\in\mathcal{X}$, and for each $\widetilde{x}\in\mathcal{X}$, the corresponding Picard sequence $\{t^n\widetilde{x}\}_{n\in\mathbb{N}}$ converges to $\widetilde{x}^*$ as $n\to+\infty$.

The quasi-contractive condition (1) of Ćirić is, in fact, the generalization of the following six well-known contractive conditions:

$$\widetilde{d}(t\widetilde{x}, t\widetilde{y}) \leq k_1 \cdot \widetilde{d}(\widetilde{x}, \widetilde{y}),$$

$$\widetilde{d}(t\widetilde{x}, t\widetilde{y}) \leq k_2 \max\left\{\widetilde{d}(\widetilde{x}, t\widetilde{x}), \widetilde{d}(\widetilde{y}, t\widetilde{y})\right\},$$

$$\widetilde{d}(t\widetilde{x}, t\widetilde{y}) \leq k_3 \max\left\{\widetilde{d}(\widetilde{x}, t\widetilde{y}), \widetilde{d}(\widetilde{y}, t\widetilde{x})\right\},$$

$$\widetilde{d}(t\widetilde{x}, t\widetilde{y}) \leq k_4 \max\left\{\widetilde{d}(\widetilde{x}, \widetilde{y}), \widetilde{d}(\widetilde{x}, t\widetilde{x}), \widetilde{d}(\widetilde{y}, t\widetilde{y})\right\},$$

$$\widetilde{d}(t\widetilde{x}, t\widetilde{y}) \leq k_5 \max\left\{\widetilde{d}(\widetilde{x}, \widetilde{y}), \frac{\widetilde{d}(\widetilde{x}, t\widetilde{x}) + \widetilde{d}(\widetilde{y}, t\widetilde{y})}{2}, \frac{\widetilde{d}(\widetilde{x}, t\widetilde{y}) + \widetilde{d}(\widetilde{y}, t\widetilde{x})}{2}\right\},$$

$$\widetilde{d}(t\widetilde{x}, t\widetilde{y}) \leq k_6 \max\left\{\widetilde{d}(\widetilde{x}, \widetilde{y}), \widetilde{d}(\widetilde{x}, t\widetilde{x}), \widetilde{d}(\widetilde{y}, t\widetilde{y}), \frac{\widetilde{d}(\widetilde{x}, t\widetilde{y}) + \widetilde{d}(\widetilde{y}, t\widetilde{x})}{2}\right\},$$

where $k_i \in [0,1), i = \overline{1,6}$.

In 1982, Istratescu [3] introduced convex contractions in the setting of metric spaces and proved the corresponding fixed-point result. He considered the following seven values: $\widetilde{d}(\widetilde{x}, \widetilde{y}), \widetilde{d}(t\widetilde{x}, t\widetilde{y}), \widetilde{d}(\widetilde{x}, t\widetilde{x}), \widetilde{d}(t\widetilde{x}, t^2\widetilde{x}), \widetilde{d}(\widetilde{y}, t\widetilde{y}), \widetilde{d}(t\widetilde{y}, t^2\widetilde{y})$ and $\widetilde{d}(t^2\widetilde{x}, t^2\widetilde{y})$ for all $\widetilde{x}, \widetilde{y} \in \mathcal{X}$. For further details, the reader is referred to papers [4–7].

Definition 1 ([3]). *A self-mapping t on $\mathcal{X}$ is said to be a convex contraction if there exist constants $a_i \geq 0, i = \overline{1,6}$ with $\sum_{i=1}^{6} a_i < 1$ such that for all $\widetilde{x}, \widetilde{y} \in \mathcal{X}$ we have*

$$\begin{aligned} \widetilde{d}\left(t^2\widetilde{x}, t^2\widetilde{y}\right) \leq\ & a_1 \cdot \widetilde{d}(\widetilde{x}, \widetilde{y}) + a_2 \cdot \widetilde{d}(t\widetilde{x}, t\widetilde{y}) + a_3 \cdot \widetilde{d}(\widetilde{x}, t\widetilde{x}) \\ & + a_4 \cdot \widetilde{d}\left(t\widetilde{x}, t^2\widetilde{x}\right) + a_5 \cdot \widetilde{d}(\widetilde{y}, t\widetilde{y}) + a_6 \cdot \widetilde{d}\left(t\widetilde{y}, t^2\widetilde{y}\right). \end{aligned} \tag{2}$$

Theorem 2 ([3]). *Let $\left(\mathcal{X}, \widetilde{d}\right)$ be a complete metric space, $t : \mathcal{X} \to \mathcal{X}$ a convex contraction. Then, t has a unique fixed point $\widetilde{x}^* \in \mathcal{X}$, and for each $\widetilde{x} \in \mathcal{X}$, the sequence $\{t^n\widetilde{x}\}_{n\in\mathbb{N}}$ converges to $\widetilde{x}^*$.*

It is not difficult to see that (2) implies the next more general contractive condition

$$\widetilde{d}\left(t^2\widetilde{x}, t^2\widetilde{y}\right) \leq k_7 \max\left\{\widetilde{d}(\widetilde{x}, \widetilde{y}), \widetilde{d}(t\widetilde{x}, t\widetilde{y}), \widetilde{d}(\widetilde{x}, t\widetilde{x}), \widetilde{d}\left(t\widetilde{x}, t^2\widetilde{x}\right), \widetilde{d}(\widetilde{y}, t\widetilde{y}), \widetilde{d}\left(t\widetilde{y}, t^2\widetilde{y}\right)\right\},$$

where $k_7 = \sum_{i=1}^{6} a_i \in [0,1)$. Recall that contraction conditions for a self-mapping t on a metric space $\left(\mathcal{X}, \widetilde{d}\right)$ usually contained, at most, the following five values: $\widetilde{d}(\widetilde{x}, \widetilde{y}), \widetilde{d}(\widetilde{x}, t\widetilde{x}), \widetilde{d}(\widetilde{y}, t\widetilde{y})$, $\widetilde{d}(\widetilde{x}, t\widetilde{y})$, and $\widetilde{d}(\widetilde{y}, t\widetilde{x})$. Recently, by adding four new values $\widetilde{d}(t^2\widetilde{x}, \widetilde{x})$, $\widetilde{d}(t^2\widetilde{x}, t\widetilde{x})$, $\widetilde{d}(t^2x, y)$, and $\widetilde{d}(t^2\widetilde{x}, t\widetilde{y})$ to a contraction condition, Kumam et al. [8] introduced a new generalization of the Ćirić fixed-point theorem [2] called a generalized quasi contraction.

Definition 2 ([8]). *Let $t : \mathcal{X} \to \mathcal{X}$ be a self-mapping on a complete metric space $\left(\mathcal{X}, \widetilde{d}\right)$ and $\lambda \in [0,1)$ such that for all $\widetilde{x}, \widetilde{y} \in \mathcal{X}$*

$$\begin{aligned} \widetilde{d}(t\widetilde{x}, t\widetilde{y}) \leq \lambda \cdot \max\Big\{ & \widetilde{d}(\widetilde{x}, \widetilde{y}), \widetilde{d}(\widetilde{x}, t\widetilde{x}), \widetilde{d}(\widetilde{y}, t\widetilde{y}), \widetilde{d}(\widetilde{x}, t\widetilde{y}), \widetilde{d}(\widetilde{y}, t\widetilde{x}), \\ & \widetilde{d}\left(t^2\widetilde{x}, \widetilde{x}\right), \widetilde{d}\left(t^2\widetilde{x}, t\widetilde{x}\right), \widetilde{d}\left(t^2\widetilde{x}, \widetilde{y}\right), \widetilde{d}\left(t^2\widetilde{x}, t\widetilde{y}\right)\Big\}. \end{aligned} \tag{3}$$

Then the mapping t is called a generalized quasi-contraction.

For this new type of contractive mapping in metric spaces, the authors in [8] proved the following result:

Theorem 3 ([8]). *Each generalized quasi-contraction t on complete metric space $\left(\mathcal{X}, \widetilde{d}\right)$ has a unique fixed point $\widetilde{x}^* \in \mathcal{X}$. Moreover, for each $\widetilde{x} \in \mathcal{X}$ the corresponding Picard sequence $\{t^n \widetilde{x}\}_{n\in\mathbb{N}}$ converges to $\widetilde{x}^*$ as $n \to +\infty$.*

D. Wardowski [9] introduced the notion of F-contraction and proved a new fixed-point theorem for it in his attempt to generalize the Banach contraction principle [1].

Definition 3 ([9]). *Let $\widetilde{\mathcal{F}}$ denote the class of all functions $\widetilde{F} : (0, +\infty) \to (-\infty, +\infty)$ which satisfy the following:*

(F1) $\widetilde{F}$ is strictly increasing, that is, for all $\widetilde{u}, \widetilde{v} \in (0, +\infty)$ if $\widetilde{u} < \widetilde{v}$ then $\widetilde{F}(\widetilde{u}) < \widetilde{F}(\widetilde{v})$;

(F2) For each sequence $\{\widetilde{a}_n\}_{n\in\mathbb{N}}$ of positive numbers holds: $\lim_{n\to+\infty} \widetilde{a}_n = 0$ if and only if $\lim_{n\to+\infty} \widetilde{F}(\widetilde{a}_n) = -\infty$;

(F3) There exists $k \in (0, 1)$ such that $\lim_{\widetilde{r}\to 0^+} \widetilde{r}^k \widetilde{F}(\widetilde{r}) = 0$.

A mapping $t : \mathcal{X} \to \mathcal{X}$ is said to be an F-contraction on $\left(\mathcal{X}, \widetilde{d}\right)$ if there exist $\widetilde{F} \in \widetilde{\mathcal{F}}$ and $\widetilde{\tau} > 0$ such that for all $\widetilde{x}, \widetilde{y} \in X$,

$$\widetilde{d}(t\widetilde{x}, t\widetilde{y}) > 0 \text{ yields } \widetilde{\tau} + \widetilde{F}\left(\widetilde{d}(t\widetilde{x}, t\widetilde{y})\right) \le \widetilde{F}\left(\widetilde{d}(\widetilde{x}, \widetilde{y})\right). \tag{4}$$

It is easy to check that the functions $\widetilde{F}_i : (0, +\infty) \to (-\infty, +\infty), i = \overline{1, 4}$, defined with $\widetilde{F}_1(\widetilde{r}) = \ln \widetilde{r}$; $\widetilde{F}_2(\widetilde{r}) = \widetilde{r} + \ln \widetilde{r}$; $\widetilde{F}_3(\widetilde{r}) = -\widetilde{r}^{-\frac{1}{2}}$; $\widetilde{F}_4(\widetilde{r}) = \ln(\widetilde{r} + \widetilde{r}^2)$, are in $\widetilde{\mathcal{F}}$.

Theorem 4 ([9]). *Let $\left(\mathcal{X}, \widetilde{d}\right)$ be a complete metric space and let $t : \mathcal{X} \to \mathcal{X}$ be an F-contraction. Then, t has a unique fixed point $\widetilde{x}^* \in \mathcal{X}$. On the other hand, the sequence $\{t^n \widetilde{x}\}_{n\in\mathbb{N}}$ converges to $\widetilde{x}^*$ for every $\widetilde{x} \in \mathcal{X}$.*

Remark 1. *Notice that Theorem 4 is true if $\widetilde{F} : (0, +\infty) \to (-\infty, +\infty)$ is non-decreasing. Moreover, if the function $\widetilde{F}$: $(0, +\infty) \to (-\infty, +\infty)$ is non-decreasing then we conclude that there are $\lim_{\widetilde{r}\to\widetilde{q}^-} \widetilde{F}(\widetilde{r}) = \widetilde{F}(\widetilde{q}-)$ and $\lim_{\widetilde{r}\to\widetilde{q}^+} \widetilde{F}(\widetilde{r}) = \widetilde{F}(\widetilde{q}+)$. For more details on monotone functions, as well as on F-contractions, see [10–20].*

Motivated by the idea of Wardowski and Dung [20], Dung and Hang [21] introduced the notion of a generalized F-contraction and proved some fixed-point theorems for such maps.

Definition 4 ([21]). *Let $\left(\mathcal{X}, \widetilde{d}\right)$ be a metric space. A mapping $t : \mathcal{X} \to \mathcal{X}$ is said to be a generalized F-contraction on $\left(\mathcal{X}, \widetilde{d}\right)$ if there exist $\widetilde{F} \in \widetilde{\mathcal{F}}$ and $\widetilde{\tau} > 0$ such that, for all $\widetilde{x}, \widetilde{y} \in \mathcal{X}$,*

$$\widetilde{d}(t\widetilde{x}, t\widetilde{y}) > 0 \text{ yields } \widetilde{\tau} + \widetilde{F}\left(\widetilde{d}(t\widetilde{x}, t\widetilde{y})\right) \le \widetilde{F}(\mathcal{N}(\widetilde{x}, \widetilde{y})), \tag{5}$$

where

$$\begin{aligned} \mathcal{N}(\widetilde{x}, \widetilde{y}) = \max\Bigg\{ & \widetilde{d}(\widetilde{x}, \widetilde{y}), \widetilde{d}(\widetilde{x}, t\widetilde{x}), \widetilde{d}(\widetilde{y}, t\widetilde{y}), \frac{\widetilde{d}(\widetilde{x}, t\widetilde{y}) + \widetilde{d}(\widetilde{y}, t\widetilde{x})}{2}, \\ & \frac{\widetilde{d}(t^2\widetilde{x}, \widetilde{x}) + \widetilde{d}(t^2\widetilde{x}, t\widetilde{y})}{2}, \widetilde{d}\left(t^2\widetilde{x}, t\widetilde{x}\right), \widetilde{d}\left(t^2\widetilde{x}, \widetilde{y}\right), \widetilde{d}\left(t^2\widetilde{x}, t\widetilde{y}\right) \Bigg\}. \end{aligned} \tag{6}$$

Theorem 5 ([21]). *Let $\left(\mathcal{X}, \widetilde{d}\right)$ be a complete metric space and let $t : \mathcal{X} \to \mathcal{X}$ be a generalized F-contraction mapping. If t or $\widetilde{F}$ is continuous, then t has a unique fixed point $\widetilde{x}^* \in \mathcal{X}$, and for every $\widetilde{x} \in \mathcal{X}$ the sequence $\{t^n \widetilde{x}\}_{n\in\mathbb{N}}$ converges to $\widetilde{x}^*$.*

In [22], the authors described a large class of functions by replacing the conditions (F2) and (F3) with the next ones:

(F2′) $\inf F = -\infty$ or, also, by

(F2″)there exists a sequence $\{\alpha_n\}_{n\in\mathbb{N}}$ of positive real numbers such that $\lim_{n\to+\infty} F(\alpha_n) = -\infty$,

(F3′) $\widetilde{F}$ is continuous on $(0, +\infty)$.

The authors in [22] denote by $\mathcal{F}$ the family of all functions $\widetilde{F} : (0, +\infty) \to (-\infty, +\infty)$ which satisfy conditions(F1), (F2′), and (F3′).

Example 1 ([22], Example 1.8). *Let $\widetilde{F}_1(\widetilde{r}) = \frac{-1}{\widetilde{r}}, \widetilde{F}_2(\widetilde{r}) = \frac{-1}{\widetilde{r}} + \widetilde{r}, \widetilde{F}_3(\widetilde{r}) = \frac{1}{1-e^{\widetilde{r}}}, \widetilde{F}_4(\widetilde{r}) = \frac{1}{e^{\widetilde{r}}-e^{-\widetilde{r}}}$. Then $\widetilde{F}_i \in \mathcal{F}$, $i = \overline{1,4}$.*

Under this new conditions, defined as the conditions (F1), (F2′), and (F3′), the authors in [22] proved some Wardowski and Suzuki–Wardowski-type fixed-point results in metric spaces as follows.

Theorem 6 ([22], Theorem 2.2). *Let t be a self-mapping of a complete metric space $\left(\mathcal{X}, \widetilde{d}\right)$. Suppose that there exist $\widetilde{F} \in \mathcal{F}$ and $\widetilde{\tau} > 0$ such that, for all $\widetilde{x}, \widetilde{y} \in \mathcal{X}$,*

$$\widetilde{d}(t\widetilde{x}, t\widetilde{y}) > 0 \text{ yields } \widetilde{\tau} + \widetilde{F}\left(\widetilde{d}(t\widetilde{x}, t\widetilde{y})\right) \le \widetilde{F}\left(\widetilde{d}(\widetilde{x}, \widetilde{y})\right). \tag{7}$$

Then, t has a unique fixed point $\widetilde{x}^* \in \mathcal{X}$, and for every $\widetilde{x} \in \mathcal{X}$ the sequence $\{t^n \widetilde{x}\}_{n\in\mathbb{N}}$ converges to $\widetilde{x}^*$.

Theorem 7 ([22]). *Let t be a self-mapping of a complete metric space $\left(\mathcal{X}, \widetilde{d}\right)$ into itself. Suppose that there exist $\widetilde{F} \in \mathcal{F}$ and $\widetilde{\tau} > 0$ such that, for all $\widetilde{x}, \widetilde{y} \in \mathcal{X}$,*

$$\frac{1}{2}\widetilde{d}(\widetilde{x}, t\widetilde{x}) < \widetilde{d}(\widetilde{x}, \widetilde{y}) \text{ yields } \widetilde{\tau} + \widetilde{F}\left(\widetilde{d}(t\widetilde{x}, t\widetilde{y})\right) \le \widetilde{F}\left(\widetilde{d}(\widetilde{x}, \widetilde{y})\right). \tag{8}$$

Then, t has a unique fixed point $\widetilde{x}^* \in \mathcal{X}$, and for every $\widetilde{x}_0 \in \mathcal{X}$ the sequence $\{t^n \widetilde{x}_0\}_{n\in\mathbb{N}}$ converges to $\widetilde{x}^*$.

In order to explain some aspects of the paper we will use the following definition.

Definition 5 ([23–25]). *Let $t : \mathcal{X} \to \mathcal{X}$ and $\widetilde{\alpha} : \mathcal{X} \times \mathcal{X} \to [0, +\infty)$. Then t is said to be triangular $\widetilde{\alpha}-$admissible if*

(t_1) $\widetilde{\alpha}(\widetilde{x}, \widetilde{y}) \ge 1$ implies $\widetilde{\alpha}(t\widetilde{x}, t\widetilde{y}) \ge 1$ for all $\widetilde{x}, \widetilde{y} \in \mathcal{X}$;

(t_2) $\widetilde{\alpha}(\widetilde{x}, \widetilde{z}) \ge 1$ and $\widetilde{\alpha}(\widetilde{z}, \widetilde{y}) \ge 1$ imply $\widetilde{\alpha}(\widetilde{x}, \widetilde{y}) \ge 1$ for all $\widetilde{x}, \widetilde{y}, \widetilde{z} \in \mathcal{X}$.

If $t : \mathcal{X} \to \mathcal{X}$ and $\widetilde{\alpha} : \mathcal{X} \times \mathcal{X} \to [0, +\infty)$ satisfy only (t_1), then t is $\widetilde{\alpha}-$admissible.

The property of the triangular alpha-admissible mapping has been given in ([24], see Lemma 7) as:

Lemma 1 ([24]). *Let t be a triangular $\widetilde{\alpha}-$admissible mapping. Assume that there exists $\widetilde{x}_0 \in \mathcal{X}$ such that $\widetilde{\alpha}(\widetilde{x}_0, t\widetilde{x}_0) \ge 1$. Define sequence $\{\widetilde{x}_n\}$ by $\widetilde{x}_n = t^n \widetilde{x}_0$. Then,*

$$\widetilde{\alpha}(\widetilde{x}_m, \widetilde{x}_n) \ge 1 \text{ for all } m, n \in \mathbb{N}\cup\{0\} \text{ with } m < n. \tag{9}$$

Definition 6 ([26]). *Let t be an $\widetilde{\alpha}-$admissible mapping on a non-empty set $\mathcal{X}$. We say that t has the property*
(H) if for each $\widetilde{x},\widetilde{y}\in Fix(t)$, there exists $\widetilde{z}\in\mathcal{X}$ such that $\widetilde{\alpha}(\widetilde{x},\widetilde{z})\geq 1$ and $\widetilde{\alpha}(\widetilde{y},\widetilde{z})\geq 1$.

Definition 7 ([26], Definition 7). *An $\widetilde{\alpha}-$admissible mapping t is said to be an $\widetilde{\alpha}^*-$admissible, if for each $\widetilde{x},\widetilde{y}\in Fix(t)\neq\emptyset$, we have $\widetilde{\alpha}(\widetilde{x},\widetilde{y})\geq 1$. If $Fix(t)=\emptyset$, we say that t is vacuously $\widetilde{\alpha}^*-$admissible.*

In [26], the authors introduced the following notion and proved two corresponding results.

Definition 8 ([26], Definition 8). *A self-mapping t on $\mathcal{X}$ is said to be an $\widetilde{\alpha}-F-$convex contraction if there exist two functions $\widetilde{\alpha}:\mathcal{X}\times\mathcal{X}\to[0,+\infty)$ and $\widetilde{F}\in\widetilde{\mathcal{F}}$ such that*

$$\widetilde{d}^p\left(t^2\widetilde{x},t^2\widetilde{y}\right)>0 \text{ yields } \widetilde{\tau}+\widetilde{F}\left(\widetilde{\alpha}(\widetilde{x},\widetilde{y})\widetilde{d}^p\left(t^2\widetilde{x},t^2\widetilde{y}\right)\right)\leq\widetilde{F}(\mathcal{M}^p(\widetilde{x},\widetilde{y})) \tag{10}$$

for all $\widetilde{x},\widetilde{y}\in\mathcal{X}$ where $p\geq 1,\widetilde{\tau}>0$ and

$$\mathcal{M}^p(\widetilde{x},\widetilde{y})=\max\left\{\widetilde{d}^p(\widetilde{x},\widetilde{y}),\ \widetilde{d}^p(t\widetilde{x},t\widetilde{y}),\ \widetilde{d}^p(\widetilde{x},t\widetilde{x}),\widetilde{d}^p\left(t\widetilde{x},t^2\widetilde{x}\right),\ \widetilde{d}^p(\widetilde{y},t\widetilde{y}),\ \widetilde{d}^p\left(t\widetilde{y},t^2\widetilde{y}\right)\right\}.$$

Remark 2. *Condition (10) is correct if $\widetilde{\alpha}:\mathcal{X}\times\mathcal{X}\to(0,+\infty)$. This is due to the definition of the function $\widetilde{F}$ because it maps $(0,+\infty)$ to $(-\infty,+\infty)$. Note that examples 6, 7 and 8 from [26] are trivial because $\widetilde{\alpha}(\widetilde{x},\widetilde{y})=1$ for all $\widetilde{x},\widetilde{y}\in\mathcal{X}$.*

Lemma 2 ([26], Lemma 1). *Let $\left(\mathcal{X},\widetilde{d}\right)$ be a metric space and $t:\mathcal{X}\to\mathcal{X}$ be an $\widetilde{\alpha}-F-$convex contraction satisfying the following conditions:*
(i) t is $\widetilde{\alpha}-$admissible;
(ii) there exists $\widetilde{x}_0\in\mathcal{X}$ such that $\widetilde{\alpha}(\widetilde{x}_0,t\widetilde{x}_0)\geq 1$.

Define a sequence $\{\widetilde{x}_n\}$ in $\mathcal{X}$ by $\widetilde{x}_{n+1}=t\widetilde{x}_n=t^{n+1}\widetilde{x}_0$ for all $n\geq 0$. Then $\left\{\widetilde{d}^p(\widetilde{x}_n,\widetilde{x}_{n+1})\right\}$ is strictly non-increasing sequence in $\mathcal{X}$.

Theorem 8 ([26], Theorem 2). *Let $\left(\mathcal{X},\widetilde{d}\right)$ be a complete metric space and $t:\mathcal{X}\to\mathcal{X}$ be an $\widetilde{\alpha}-F-$convex contraction satisfying the following conditions:*
(i) t is $\widetilde{\alpha}-$admissible;
(ii) There exists $\widetilde{x}_0\in\mathcal{X}$ such that $\widetilde{\alpha}(\widetilde{x}_0,t\widetilde{x}_0)\geq 1$;
(iii) t is continuous or orbital continuous on $\mathcal{X}$.

Then, t has a fixed point in $\mathcal{X}$. Further, if t is $\widetilde{\alpha}^*-$admissible, then t has a unique fixed point $\widetilde{z}\in\mathcal{X}$. Moreover, for any $\widetilde{x}_0\in\mathcal{X}$ if $\widetilde{x}_{n+1}=t^{n+1}\widetilde{x}_0\neq t\widetilde{x}_n$ for all $n\in\mathbb{N}\cup\{0\}$, then $\lim_{n\to+\infty}t^n\widetilde{x}_0=\widetilde{z}$.

Let us introduce two well-known lemmas [27] which we will further use in the proofs of our results:

Lemma 3. *Suppose that $\{\widetilde{x}_n\}_{n\in\mathbb{N}}$ in a metric space $\left(\mathcal{X},\widetilde{d}\right)$ that satisfies $\lim_{n\to+\infty}\widetilde{d}(\widetilde{x}_n,\widetilde{x}_{n+1})=0$ is not a Cauchy sequence. Therefore, there exist $\widetilde{\eta}>0$ and sequences of positive integers $\{n_k\},\{m_k\},n_k>m_k>k$ such that each of the next sequences*

$$\widetilde{d}(\widetilde{x}_{n_k},\widetilde{x}_{m_k}),\ \widetilde{d}(\widetilde{x}_{n_k+1},\widetilde{x}_{m_k}),\ \widetilde{d}(\widetilde{x}_{n_k},\widetilde{x}_{m_k-1}),\ \widetilde{d}(\widetilde{x}_{n_k+1},\widetilde{x}_{m_k-1}),\ \widetilde{d}(\widetilde{x}_{n_k+1},\widetilde{x}_{m_k+1})$$

tends to $\widetilde{\eta}^+$ when $k\to+\infty$.

Lemma 4. *Let* $\{\widetilde{x}_{n+1}\} = \{t\widetilde{x}_n\} = \{t^n\widetilde{x}_0\}, n \in \mathbb{N}\cup\{0\}, t^0\widetilde{x}_0 = \widetilde{x}_0$ *be a Picard sequence in a metric space* $\left(\mathcal{X}, \widetilde{d}\right)$ *induced by a mapping* $t : \mathcal{X} \to \mathcal{X}$ *and* $\widetilde{x}_0 \in \mathcal{X}$ *be an initial point. If* $\widetilde{d}(\widetilde{x}_n, \widetilde{x}_{n+1}) < \widetilde{d}(\widetilde{x}_{n-1}, \widetilde{x}_n)$ *for all* $n \in \mathbb{N}$ *then* $\widetilde{x}_n \neq \widetilde{x}_m$ *whenever* $n \neq m$.

In 1974, Ćirić [2] (see also [28]) introduced the notion of orbital continuity. If t is a self-mapping of a metric space $(\mathcal{X}; \widetilde{d})$, then the set $O(\widetilde{x}; t) = \{t^n\widetilde{x} : n \in \mathbb{N} \cup \{0\}\}$ is called the orbit of t at $\widetilde{x}$, and t is called orbital continuous if $\widetilde{u} = \lim_{i\to+\infty} tt^{m_i}\widetilde{x}$. Continuity of t obviously implies orbital continuity, but the converse is not necessarily true [28]. This notion was used by the authors of [26].

2. Improved Results

Our first result in this section is the new proof of Theorem 5 (main result from [21]). In it we will use only the property (F1) of the function $\widetilde{F} : (0, +\infty) \to (-\infty, +\infty)$.

Proof of Theorem 5. First, from (5), whenever $\widetilde{x} \neq \widetilde{y}$, this yields

$$\widetilde{d}(t\widetilde{x}, t\widetilde{y}) < \mathcal{N}(\widetilde{x}, \widetilde{y}), \tag{11}$$

where

$$\mathcal{N}(\widetilde{x}, \widetilde{y}) = \max\left\{ \widetilde{d}(\widetilde{x}, \widetilde{y}),\, \widetilde{d}(\widetilde{x}, t\widetilde{x}),\, \widetilde{d}(\widetilde{y}, t\widetilde{y}),\, \frac{\widetilde{d}(\widetilde{x}, t\widetilde{y}) + \widetilde{d}(\widetilde{y}, t\widetilde{x})}{2}, \right.$$
$$\left. \frac{\widetilde{d}(t^2\widetilde{x}, \widetilde{x}) + \widetilde{d}(t^2\widetilde{x}, t\widetilde{y})}{2}, \widetilde{d}\left(t^2\widetilde{x}, t\widetilde{x}\right), \widetilde{d}\left(t^2\widetilde{x}, \widetilde{y}\right), \widetilde{d}\left(t^2\widetilde{x}, t\widetilde{y}\right) \right\}.$$

First, we prove the uniqueness of a possible fixed point of the mapping t. Indeed, if $\widetilde{x}$ and $\widetilde{y}$ are two distinct fixed point of t, we get:

$$\begin{aligned} \widetilde{d}(t\widetilde{x}, t\widetilde{y}) = \widetilde{d}(\widetilde{x}, \widetilde{y}) &< \max\left\{ \widetilde{d}(\widetilde{x}, \widetilde{y}), 0, 0, \frac{\widetilde{d}(\widetilde{x}, \widetilde{y}) + \widetilde{d}(\widetilde{y}, \widetilde{x})}{2}, \frac{0 + \widetilde{d}(\widetilde{x}, \widetilde{y})}{2}, 0, \widetilde{d}(\widetilde{x}, \widetilde{y}), \widetilde{d}(\widetilde{x}, \widetilde{y}) \right\} \\ &= \widetilde{d}(\widetilde{x}, \widetilde{y}) \end{aligned} \tag{12}$$

which is a contradiction.

Next, we show the existence of a fixed point for the mapping t. For this, let $\widetilde{x}_0$ be an arbitrary point from $\mathcal{X}$ and $\{\widetilde{x}_n\}_{n\in\mathbb{N}}$ be a corresponding Picard sequence. If $\widetilde{x}_q = \widetilde{x}_{q+1}$ for some $q \in \mathbb{N}$, then according to the previous work shown,$\widetilde{x}_q$ is a unique fixed point of t. Therefore, let $\widetilde{x}_n \neq \widetilde{x}_{n+1}$ for all $n \in \mathbb{N} \cup \{0\}$. Putting $\widetilde{x} = \widetilde{x}_{n-1}$ and $\widetilde{y} = \widetilde{x}_n$ in (11), we obtain

$$\begin{aligned} \widetilde{d}(\widetilde{x}_n, \widetilde{x}_{n+1}) &< \max\Big\{ \widetilde{d}(\widetilde{x}_{n-1}, \widetilde{x}_n), \widetilde{d}(\widetilde{x}_{n-1}, \widetilde{x}_n), \widetilde{d}(\widetilde{x}_n, \widetilde{x}_{n+1}), \\ &\qquad \frac{\widetilde{d}(\widetilde{x}_{n-1}, \widetilde{x}_{n+1}) + 0}{2}, \frac{\widetilde{d}(\widetilde{x}_{n+1}, \widetilde{x}_{n-1}) + 0}{2}, 0, \widetilde{d}(\widetilde{x}_{n+1}, \widetilde{x}_n), 0 \Big\} \\ &= \max\left\{ \widetilde{d}(\widetilde{x}_{n-1}, \widetilde{x}_n), \widetilde{d}(\widetilde{x}_n, \widetilde{x}_{n+1}), \frac{\widetilde{d}(\widetilde{x}_{n-1}, \widetilde{x}_n) + \widetilde{d}(\widetilde{x}_n, \widetilde{x}_{n+1})}{2}, \widetilde{d}(\widetilde{x}_{n+1}, \widetilde{x}_n) \right\} \\ &\leq \max\Big\{ \widetilde{d}(\widetilde{x}_{n-1}, \widetilde{x}_n), \widetilde{d}(\widetilde{x}_n, \widetilde{x}_{n+1}) \Big\}. \end{aligned} \tag{13}$$

Note that (13) implies that $\widetilde{d}(\widetilde{x}_n, \widetilde{x}_{n+1}) < \widetilde{d}(\widetilde{x}_{n-1}, \widetilde{x}_n)$, because otherwise we have a contradiction. Therefore, there exists $\widetilde{\xi} \geq 0$ such that $\widetilde{d}(\widetilde{x}_n, \widetilde{x}_{n+1}) \to \widetilde{\xi}$ as $n \to +\infty$. Note that condition (13) also implies that

$$\widetilde{\tau} + \widetilde{F}\left(\widetilde{d}(\widetilde{x}_n, \widetilde{x}_{n+1})\right) \leq \widetilde{F}\left(\widetilde{d}(\widetilde{x}_{n-1}, \widetilde{x}_n)\right), \tag{14}$$

that is, by taking the limit in (14), we get according to Remark 1 : $\widetilde{\tau} + \widetilde{F}\left(\widetilde{\xi}+\right) \leq \widetilde{F}\left(\widetilde{\xi}+\right)$, which is a contradiction with $\widetilde{\tau} > 0$. In order to prove that the sequence $\{\widetilde{x}_n\}_{n\in\mathbf{N}}$ is a

Cauchy, we will use Lemma 3. Namely, putting $\widetilde{x} = \widetilde{x}_{n_k}$ and $\widetilde{y} = \widetilde{x}_{m_k}$ in (5), we get the following inequality:

$$\widetilde{\tau} + \widetilde{F}\left(\widetilde{d}(\widetilde{x}_{n_k+1}, \widetilde{x}_{m_k+1})\right) \leq \widetilde{F}(\mathcal{N}(\widetilde{x}_{n_k}, \widetilde{x}_{m_k})), \tag{15}$$

where

$$\begin{aligned}\mathcal{N}(\widetilde{x}_{n_k}, \widetilde{x}_{m_k}) = \max\Big\{ & \widetilde{d}(\widetilde{x}_{n_k}, \widetilde{x}_{m_k}), \widetilde{d}(\widetilde{x}_{n_k}, \widetilde{x}_{n_k+1}), \widetilde{d}(\widetilde{x}_{m_k}, \widetilde{x}_{m_k+1}), \\ & \frac{\widetilde{d}(\widetilde{x}_{n_k}, \widetilde{x}_{m_k+1}) + \widetilde{d}(\widetilde{x}_{m_k}, \widetilde{x}_{n_k+1})}{2}, \quad \frac{\widetilde{d}(\widetilde{x}_{n_k+2}, \widetilde{x}_{n_k}) + \widetilde{d}(\widetilde{x}_{n_k+2}, \widetilde{x}_{m_k+1})}{2}, \\ & \widetilde{d}(\widetilde{x}_{n_k+2}, \widetilde{x}_{n_k+1}), \widetilde{d}(\widetilde{x}_{n_k+2}, \widetilde{x}_{m_k}), \widetilde{d}(\widetilde{x}_{n_k+2}, \widetilde{x}_{m_k+1}) \Big\}.\end{aligned}$$

Hence, $\mathcal{N}(\widetilde{x}_{n_k}, \widetilde{x}_{m_k}) \to \max\left\{\widetilde{\eta}, 0, 0, \frac{\widetilde{\eta}+\widetilde{\eta}}{2}, \frac{0+\widetilde{\eta}}{2}, 0, \widetilde{\eta}, \widetilde{\eta}\right\} = \widetilde{\eta}$ as $k \to +\infty$. Further, (15) yields $\widetilde{\tau} + \widetilde{F}(\widetilde{\eta}+) \leq \widetilde{F}(\widetilde{\eta}+)$, which is a contradiction with $\widetilde{\tau} > 0$.
The completeness of metric space $\left(\mathcal{X}, \widetilde{d}\right)$ implies that $\widetilde{x}_n$ converges to some $\widetilde{x} \in \mathcal{X}$. If the mapping t is continuous, we get

$$\widetilde{x} = \lim_{n\to+\infty} \widetilde{x}_n = \lim_{n\to+\infty} t\widetilde{x}_{n-1} = t\left(\lim_{n\to+\infty} \widetilde{x}_{n-1}\right) = t\widetilde{x},$$

that is, $\widetilde{x}$ is a unique fixed point of t. Now, suppose that the mapping $\widetilde{F}$ is continuous. Put $\widetilde{x} = \widetilde{x}_n$ and $\widetilde{y} = \widetilde{x}$ in (5). Since $\widetilde{x}_n \neq \widetilde{x}_m$ whenever $n \neq m$, we can suppose that there is $k \in \mathbb{N}$ such that $\widetilde{x} \notin \{\widetilde{x}_n \text{ for } n > k\}$. This means that $\widetilde{d}(t\widetilde{x}_n, t\widetilde{x}) > 0$ for $n > k$. Then we obtain

$$\widetilde{d}(t\widetilde{x}_n, t\widetilde{x}) > 0 \text{ yields } \widetilde{\tau} + \widetilde{F}\left(\widetilde{d}(t\widetilde{x}_n, t\widetilde{x})\right) \leq \widetilde{F}(\mathcal{N}(\widetilde{x}_n, \widetilde{x})), \tag{16}$$

where

$$\begin{aligned}\mathcal{N}(\widetilde{x}_n, \widetilde{x}) = \max\Bigg\{ & \widetilde{d}(\widetilde{x}_n, \widetilde{x}), \widetilde{d}(\widetilde{x}_n, t\widetilde{x}_n), \widetilde{d}(\widetilde{x}, t\widetilde{x}), \frac{\widetilde{d}(\widetilde{x}_n, t\widetilde{x}) + \widetilde{d}(\widetilde{x}, t\widetilde{x})}{2}, \\ & \frac{\widetilde{d}(t^2\widetilde{x}_n, \widetilde{x}) + \widetilde{d}(t^2\widetilde{x}_n, t\widetilde{x})}{2}, \widetilde{d}\left(t^2\widetilde{x}_n, t\widetilde{x}_n\right), \widetilde{d}\left(t^2\widetilde{x}_n, \widetilde{x}\right), \widetilde{d}\left(t^2\widetilde{x}_n, t\widetilde{x}\right) \Bigg\};\end{aligned}$$

that is,

$$\begin{aligned}\mathcal{N}(\widetilde{x}_n, \widetilde{x}) = \max\Big\{ & \widetilde{d}(\widetilde{x}_n, \widetilde{x}), \widetilde{d}(\widetilde{x}_n, \widetilde{x}_{n+1}), \widetilde{d}(\widetilde{x}, t\widetilde{x}), \frac{\widetilde{d}(\widetilde{x}_n, t\widetilde{x}) + \widetilde{d}(\widetilde{x}, t\widetilde{x})}{2}, \\ & \frac{\widetilde{d}(\widetilde{x}_{n+2}, \widetilde{x}) + \widetilde{d}(\widetilde{x}_{n+2}, t\widetilde{x})}{2}, \widetilde{d}(\widetilde{x}_{n+2}, \widetilde{x}_{n+1}), \widetilde{d}(\widetilde{x}_{n+2}, \widetilde{x}), \widetilde{d}(\widetilde{x}_{n+2}, t\widetilde{x}) \Big\}.\end{aligned} \tag{17}$$

Letting $n \to +\infty$ we get that $\mathcal{N}(\widetilde{x}_n, \widetilde{x}) \to \max\left\{0, 0, \widetilde{d}(\widetilde{x}, t\widetilde{x}), \frac{\widetilde{d}(\widetilde{x}, t\widetilde{x})}{2}, 0, 0, \widetilde{d}(\widetilde{x}, t\widetilde{x})\right\} = \widetilde{d}(\widetilde{x}, t\widetilde{x})$. We used the next inequalities: $\widetilde{d}(\widetilde{x}_n, t\widetilde{x}) \leq \widetilde{d}(\widetilde{x}_n, \widetilde{x}) + \widetilde{d}(\widetilde{x}, t\widetilde{x})$ and $\widetilde{d}(\widetilde{x}_{n+2}, t\widetilde{x}) \leq \widetilde{d}(\widetilde{x}_{n+2}, \widetilde{x}) + \widetilde{d}(\widetilde{x}, t\widetilde{x})$. If $\widetilde{d}(\widetilde{x}, t\widetilde{x}) > 0$, then (16) implies

$$\widetilde{\tau} + \widetilde{F}\left(\widetilde{d}(\widetilde{x}, t\widetilde{x})\right) \leq \widetilde{F}\left(\widetilde{d}(\widetilde{x}, t\widetilde{x})\right),$$

which is a contradiction. Hence, $\widetilde{x}$ is a unique fixed point of t in both cases: t or $\widetilde{F}$ is continuous. □

Remark 3. *Since $\widetilde{d}(\widetilde{x}, \widetilde{y}) \leq \mathcal{N}(\widetilde{x}, \widetilde{y})$ for all $\widetilde{x}, \widetilde{y} \in \mathcal{X}$, we have that Theorem 5 (that is, Theorem 2.1 from [22]) is an immediate consequence of Theorem 4.*

Our second completely new result in this paper is the proof of Theorem 6. We will use only the property (F1) of the given mapping $\widetilde{F} : (0, +\infty) \to (-\infty, +\infty)$ which means improvement of the corresponding approach in [22].

Proof of Theorem 6. First, (8) implies the uniqueness of a possible fixed point of the mapping $t : \mathcal{X} \to \mathcal{X}$. Indeed, if $\widetilde{x}$ and $\widetilde{y}$ are two distinct fixed points of t, then because

$$\frac{1}{2}\widetilde{d}(\widetilde{x}, t\widetilde{x}) = \frac{1}{2}\widetilde{d}(\widetilde{x}, \widetilde{x}) = 0 < \widetilde{d}(\widetilde{x}, \widetilde{y}),$$

we have that

$$\widetilde{\tau} + \widetilde{F}\left(\widetilde{d}(t\widetilde{x}, t\widetilde{y})\right) \leq \widetilde{F}\left(\widetilde{d}(\widetilde{x}, \widetilde{y})\right);$$

that is,

$$\widetilde{\tau} + \widetilde{F}\left(\widetilde{d}(\widetilde{x}, \widetilde{y})\right) \leq \widetilde{F}\left(\widetilde{d}(\widetilde{x}, \widetilde{y})\right),$$

which is a contradiction with $\widetilde{\tau} > 0$. Hence, if t has a fixed point, it is unique.

Now, we prove the existence of a fixed point for t. Let $\widetilde{x}_0 \in \mathcal{X}$ be an arbitrary point and $\{\widetilde{x}_n\}_{n\in\mathbb{N}}$ be the corresponding Picard sequence. If $\widetilde{x}_q = \widetilde{x}_{q+1}$ for some $q \in \mathbb{N}$, then $\widetilde{x}_q$ is a unique fixed point of t. Suppose that $\widetilde{x}_n \neq \widetilde{x}_{n+1}$ for each $n \in \mathbb{N}$. Since $\frac{1}{2}\widetilde{d}(\widetilde{x}_n, T\widetilde{x}_n) = \frac{1}{2}\widetilde{d}(\widetilde{x}_n, \widetilde{x}_{n+1}) < \widetilde{d}(\widetilde{x}_n, \widetilde{x}_{n+1})$, then according to (8), we get

$$\widetilde{\tau} + \widetilde{F}\left(\widetilde{d}(\widetilde{x}_n, \widetilde{x}_{n+1})\right) \leq \widetilde{F}\left(\widetilde{d}(\widetilde{x}_{n-1}, \widetilde{x}_n)\right), \tag{18}$$

for all $n \in \mathbb{N}$. This further means (because (F1)) $\widetilde{d}(\widetilde{x}_n, \widetilde{x}_{n+1}) < \widetilde{d}(\widetilde{x}_{n-1}, \widetilde{x}_n)$, that is, $\widetilde{d}(\widetilde{x}_n, \widetilde{x}_{n+1}) \to \widetilde{\delta}^*$ for some $\widetilde{\delta}^* \geq 0$. If $\widetilde{\delta}^* > 0$, we get from (18)

$$\widetilde{\tau} + \widetilde{F}\left(\widetilde{\delta}^* +\right) \leq \widetilde{F}\left(\widetilde{\delta}^* +\right),$$

which is a contradiction with $\widetilde{\tau} > 0$. In order to prove that $\{\widetilde{x}_n\}$ is a Cauchy sequence, we put $\widetilde{x} = \widetilde{x}_{n_k}$ and $\widetilde{y} = \widetilde{x}_{m_k}$ in (8). This further means that

$$\widetilde{\tau} + \widetilde{F}\left(\widetilde{d}(\widetilde{x}_{n_k+1}, \widetilde{x}_{m_k+1})\right) \leq \widetilde{F}\left(\widetilde{d}(\widetilde{x}_{n_k}, \widetilde{x}_{m_k})\right),$$

from which as $k \to +\infty$ it follows

$$\widetilde{\tau} + \widetilde{F}(\widetilde{\eta}+) \leq \widetilde{F}(\widetilde{\eta}+),$$

a contradiction with $\widetilde{\tau} > 0$.

Since $\left(\mathcal{X}, \widetilde{d}\right)$ is complete, there exists $\widetilde{x}^*$ such that $\{\widetilde{x}_n\}$ converges to $\widetilde{x}^*$, that is, $\lim_{n\to+\infty} \widetilde{d}(\widetilde{x}_n, \widetilde{x}^*) = 0$.

By using only condition (F1), one can prove, as in [22], that for all $n \in \mathbb{N}$:

$$\frac{1}{2}\widetilde{d}(\widetilde{x}_n, t\widetilde{x}_n) < \widetilde{d}(\widetilde{x}_n, \widetilde{x}^*) \text{ or } \frac{1}{2}\widetilde{d}\left(t\widetilde{x}_n, t^2\widetilde{x}_n\right) < \widetilde{d}(t\widetilde{x}_n, \widetilde{x}^*). \tag{19}$$

Now, from (19), for all $n \in \mathbb{N}$, either

$$\widetilde{\tau} + \widetilde{F}\left(\widetilde{d}(t\widetilde{x}_n, t\widetilde{x}^*)\right) < \widetilde{F}\left(\widetilde{d}(\widetilde{x}_n, \widetilde{x}^*)\right) \tag{20}$$

or

$$\widetilde{\tau} + \widetilde{F}\left(\widetilde{d}\left(t^2\widetilde{x}_n, t\widetilde{x}^*\right)\right) < \widetilde{F}\left(\widetilde{d}(t\widetilde{x}_n, \widetilde{x}^*)\right) \tag{21}$$

holds. Both the conditions (20) and (21) imply that $\widetilde{x}_n$ converges to $t\widetilde{x}^*$ as $n \to +\infty$. Because $\widetilde{x}_n$ also converges to $\widetilde{x}^*$, we obtain that $\widetilde{x}^*$ is a unique fixed point of t. The proof of Theorem 6 is complete. □

Our next new result in the sequel of this part will be the improvement of Lemma 2 (that is Lemma 1 from [26]). In fact, we will prove that the sequence $\left\{\widetilde{d}^p(\widetilde{x}_n,\widetilde{x}_{n+1})\right\}$ is strictly decreasing whenever $\widetilde{x}_n \neq \widetilde{x}_{n+1}$ for all $n \in \mathbb{N}\cup\{0\}$.

Proof of Lemma 2. If $\widetilde{x}_0 \in \mathcal{X}$ is an arbitrary point, then as in the proof of Lemma 1 in [26], it follows that $\widetilde{\alpha}(\widetilde{x}_n,\widetilde{x}_{n+1}) \geq 1$ for all $n \in \mathbb{N}\cup\{0\}$. Further, suppose that $\widetilde{x}_n \neq \widetilde{x}_{n+1}$ for all $n \in \mathbb{N}\cup\{0\}$. Putting $\widetilde{x} = \widetilde{x}_{n-1}$ and $\widetilde{y} = \widetilde{x}_n$ in (10), we get

$$\widetilde{\tau} + \widetilde{F}\Big(\widetilde{\alpha}(\widetilde{x}_{n-1},\widetilde{x}_n)\cdot\widetilde{d}^p(\widetilde{x}_{n+1},\widetilde{x}_{n+2})\Big) \leq \widetilde{F}(\mathcal{M}^p(\widetilde{x}_{n-1},\widetilde{x}_n)),$$

where

$$\begin{aligned}\mathcal{M}^p(\widetilde{x}_{n-1},\widetilde{x}_n) = \max\Big\{&\widetilde{d}^p(\widetilde{x}_{n-1},\widetilde{x}_n),\widetilde{d}^p(\widetilde{x}_n,\widetilde{x}_{n+1}),\widetilde{d}^p(\widetilde{x}_{n-1},\widetilde{x}_n),\\ &\widetilde{d}^p(\widetilde{x}_n,\widetilde{x}_{n+1}),\widetilde{d}^p(\widetilde{x}_n,\widetilde{x}_{n+1}),\widetilde{d}^p(\widetilde{x}_{n+1},\widetilde{x}_{n+2})\};\end{aligned} \tag{22}$$

that is,

$$\mathcal{M}^p(\widetilde{x}_{n-1},\widetilde{x}_n) = \max\Big\{\widetilde{d}^p(\widetilde{x}_{n-1},\widetilde{x}_n),\widetilde{d}^p(\widetilde{x}_n,\widetilde{x}_{n+1}),\widetilde{d}^p(\widetilde{x}_{n+1},\widetilde{x}_{n+2})\Big\}.$$

Finally, we get

$$\begin{aligned}&\widetilde{\tau} + \widetilde{F}\Big(\widetilde{\alpha}(\widetilde{x}_{n-1},\widetilde{x}_n)\cdot\widetilde{d}^p(\widetilde{x}_{n+1},\widetilde{x}_{n+2})\Big)\\ &\quad\leq \widetilde{F}\Big(\max\Big\{\widetilde{d}^p(\widetilde{x}_{n-1},\widetilde{x}_n),\widetilde{d}^p(\widetilde{x}_n,\widetilde{x}_{n+1}),\widetilde{d}^p(\widetilde{x}_{n+1},\widetilde{x}_{n+2})\Big\}\Big),\end{aligned} \tag{23}$$

for all $n \in \mathbb{N}\cup\{0\}$. Since $\widetilde{\alpha}(\widetilde{x}_{n-1},\widetilde{x}_n) \geq 1$, for all $n \in \mathbb{N}$ we identify that (23) becomes

$$\begin{aligned}&\widetilde{\tau} + \widetilde{F}\Big(\widetilde{d}^p(\widetilde{x}_{n+1},\widetilde{x}_{n+2})\Big)\\ &\quad\leq \widetilde{F}\Big(\max\Big\{\widetilde{d}^p(\widetilde{x}_{n-1},\widetilde{x}_n),\widetilde{d}^p(\widetilde{x}_n,\widetilde{x}_{n+1}),\widetilde{d}^p(\widetilde{x}_{n+1},\widetilde{x}_{n+2})\Big\}\Big).\end{aligned}$$

According to the property (F1) of the function $\widetilde{F}$, it follows that

$$\widetilde{\tau} + \widetilde{F}\Big(\widetilde{d}^p(\widetilde{x}_{n+1},\widetilde{x}_{n+2})\Big) \leq \widetilde{F}\Big(\max\Big\{\widetilde{d}^p(\widetilde{x}_{n-1},\widetilde{x}_n),\widetilde{d}^p(\widetilde{x}_n,\widetilde{x}_{n+1})\Big\}\Big); \tag{24}$$

that is,

$$\widetilde{d}^p(\widetilde{x}_{n+1},\widetilde{x}_{n+2}) < \max\Big\{\widetilde{d}^p(\widetilde{x}_{n-1},\widetilde{x}_n),\widetilde{d}^p(\widetilde{x}_n,\widetilde{x}_{n+1})\Big\}$$

for all $n \in \mathbb{N}$. In order to prove that $\widetilde{d}^p(\widetilde{x}_{n+1},\widetilde{x}_{n+2}) < \widetilde{d}^p(\widetilde{x}_n,\widetilde{x}_{n+1})$ for all $n \in \mathbb{N}\cup\{0\}$, we will distinguish three cases:

I. $\mathbb{N}_= = \Big\{n \in \mathbb{N} : \widetilde{d}^p(\widetilde{x}_{n-1},\widetilde{x}_n) = \widetilde{d}^p(\widetilde{x}_n,\widetilde{x}_{n+1})\Big\}$;

II. $\mathbb{N}_< = \Big\{n \in \mathbb{N} : \widetilde{d}^p(\widetilde{x}_{n-1},\widetilde{x}_n) < \widetilde{d}^p(\widetilde{x}_n,\widetilde{x}_{n+1})\Big\}$;

III. $\mathbb{N}_> = \Big\{n \in \mathbb{N} : \widetilde{d}^p(\widetilde{x}_{n-1},\widetilde{x}_n) > \widetilde{d}^p(\widetilde{x}_n,\widetilde{x}_{n+1})\Big\}$.

Obviously, $\mathbb{N} = \mathbb{N}_= \cup \mathbb{N}_< \cup \mathbb{N}_>$ and $\mathbb{N}_= \cap \mathbb{N}_< = \mathbb{N}_= \cap \mathbb{N}_> = \mathbb{N}_< \cap \mathbb{N}_> = \emptyset$. Analyzing each of these three cases, we conclude that $\widetilde{d}^p(\widetilde{x}_n,\widetilde{x}_{n+1}) < \widetilde{d}^p(\widetilde{x}_{n-1},\widetilde{x}_n)$ for all $n \in \mathbb{N}$. This further means that there exists $\lim_{n\to+\infty}\widetilde{d}^p(\widetilde{x}_n,\widetilde{x}_{n+1}) = \widetilde{\delta} \geq 0$. If $\widetilde{\delta} > 0$, then from (24), it yields

$$\widetilde{\tau} + \widetilde{F}(\widetilde{\delta}+) \leq \widetilde{F}(\widetilde{\delta}+),$$

which is a contradiction with $\widetilde{\tau} > 0$. We have proved Lemma 2. □

Our second new result is the proof of Theorem 8 (that is Theorem 2 from [26]). In it, we will use only the property (F1).

Proof of Theorem 8. As in [26] (see the proof on page 6), we get that $\widetilde{\alpha}(\widetilde{x}_n,\widetilde{x}_{n+1}) \geq 1$ for all $n \in \mathbb{N}\cup\{0\}$, where $\widetilde{x}_{n+1} = t^{n+1}\widetilde{x}_0 \neq t^n\widetilde{x}_0$ is a Picard sequence induced by any point $\widetilde{x}_0 \in \mathcal{X}$. According to the above Lemma, the sequence $\{\widetilde{x}_n\}_{n\in\mathbb{N}\cup\{0\}}$ satisfies $\widetilde{d}^p(\widetilde{x}_{n+1},\widetilde{x}_{n+2}) <$

$\widetilde{d}^p(\widetilde{x}_n, \widetilde{x}_{n+1})$ for all $n \in \mathbb{N}\cup\{0\}$ and $\widetilde{d}^p(\widetilde{x}_n, \widetilde{x}_{n+1}) \to 0$ as $n \to +\infty$. This further means that $\widetilde{d}(\widetilde{x}_n, \widetilde{x}_{n+1}) \to 0$ as $n \to +\infty$. Therefore, we can use Lemma 3. Indeed, putting $\widetilde{x} = \widetilde{x}_{n_k}$ and $\widetilde{y} = \widetilde{x}_{m_k}$ in (10) (it is clear that $\widetilde{d}^p(t^2\widetilde{x}, t^2\widetilde{y}) = \widetilde{d}^p(\widetilde{x}_{n_k+2}, \widetilde{x}_{m_k+2}) > 0$), we get

$$\widetilde{\tau} + \widetilde{F}\left(\alpha(\widetilde{x}_{n_k}, \widetilde{x}_{m_k}) \cdot \widetilde{d}^p\left(t^2\widetilde{x}_{n_k}, t^2\widetilde{x}_{m_k}\right)\right) \leq \widetilde{F}(\mathcal{M}^p(\widetilde{x}_{n_k}, \widetilde{x}_{m_k})), \tag{25}$$

where

$$\begin{aligned}\mathcal{M}^p(\widetilde{x}_{n_k}, \widetilde{x}_{m_k}) = \max\Big\{&\widetilde{d}^p(\widetilde{x}_{n_k}, \widetilde{x}_{m_k}), \widetilde{d}^p(\widetilde{x}_{n_k+1}, \widetilde{x}_{m_k+1}), \widetilde{d}^p(\widetilde{x}_{n_k}, \widetilde{x}_{n_k+1}),\\ &\widetilde{d}^p(\widetilde{x}_{n_k+1}, \widetilde{x}_{n_k+2}), \widetilde{d}^p(\widetilde{x}_{m_k}, \widetilde{x}_{m_k+1}), \widetilde{d}^p(\widetilde{x}_{m_k+1}, \widetilde{x}_{m_k+2})\Big\}\\ &\to \max\{\widetilde{\eta}^+, \widetilde{\eta}^+, 0, 0, 0, 0\} = \widetilde{\eta}^+ \text{ as } k \to +\infty.\end{aligned} \tag{26}$$

Since, according to Lemma 1, $\widetilde{\alpha}(\widetilde{x}_{n_k}, \widetilde{x}_{m_k}) \geq 1$, we identify that (25) becomes

$$\widetilde{\tau} + \widetilde{F}\left(\widetilde{d}^p(\widetilde{x}_{n_k+2}, \widetilde{x}_{m_k+2})\right) \leq \widetilde{F}(\mathcal{M}^p(\widetilde{x}_{n_k}, \widetilde{x}_{m_k})).$$

Taking the limit as $k \to +\infty$ and using Remark 1 and Lemma 3, we get

$$\widetilde{\tau} + \widetilde{F}(\widetilde{\eta}^+ +) \leq \widetilde{F}(\widetilde{\eta}^+ +),$$

which is a contradiction with $\widetilde{\tau} > 0$. Hence, the sequence $\{\widetilde{x}_n\}$ is a Cauchy. In the sequel, the proof is as in [26]. □

In the sequel, we give some immediate corollaries of Theorem 8. Addditionally, we improve two things from [26] (we consider interval $(0, +\infty)$ instead of $[0, +\infty)$, and we notice that it is sufficient to suppose that t is orbitally continuous).

Corollary 1. *Let $\left(\mathcal{X}, \widetilde{d}\right)$ be a complete metric space and $\widetilde{\alpha} : \mathcal{X} \times \mathcal{X} \to (0, +\infty)$ be a function. Suppose that $t : \mathcal{X} \to \mathcal{X}$ is a self-mapping satisfying the following conditions:*

(i) There exists $k \in [0, 1)$ such that for all $\widetilde{x}, \widetilde{y} \in \mathcal{X}$

$$\widetilde{\alpha}(\widetilde{x}, \widetilde{y}) \cdot \widetilde{d}\left(t^2\widetilde{x}, t^2\widetilde{y}\right) \leq k \cdot \mathcal{M}^1(\widetilde{x}, \widetilde{y}), \tag{27}$$

where

$$\mathcal{M}^1(\widetilde{x}, \widetilde{y}) = \max\left\{\widetilde{d}(\widetilde{x}, \widetilde{y}), \widetilde{d}(t\widetilde{x}, t\widetilde{y}), \widetilde{d}(\widetilde{x}, t\widetilde{x}), \widetilde{d}\left(t\widetilde{x}, t^2\widetilde{x}\right), \widetilde{d}(\widetilde{y}, t\widetilde{y}), \widetilde{d}\left(t\widetilde{y}, t^2\widetilde{y}\right)\right\}$$

.

(ii) t is $\widetilde{\alpha}-$admissible;

(iii) There exists $\widetilde{x}_0 \in \mathcal{X}$ such that $\widetilde{\alpha}(\widetilde{x}_0, t\widetilde{x}_0) \geq 1$;

(iv) t is orbitally continuous on $\left(\mathcal{X}, \widetilde{d}\right)$.

Then, t has a fixed point in $\mathcal{X}$. Further, if t is $\widetilde{\alpha}^*$-admissible, then t has a unique fixed point $\widetilde{z} \in \mathcal{X}$. Moreover, for any $\widetilde{x}_0 \in \mathcal{X}$ if $\widetilde{x}_{n+1} = t^{n+1}\widetilde{x}_0 \neq t^n\widetilde{x}_0$ for all $n \in \mathbb{N}\cup\{0\}$, then $\lim_{n\to+\infty} t^n\widetilde{x}_0 = \widetilde{z}$.

Corollary 2. *Let $\left(\mathcal{X}, \widetilde{d}\right)$ be a complete metric space and $\widetilde{\alpha} : \mathcal{X} \times \mathcal{X} \to (0, +\infty)$ be a function. Suppose that $t : \mathcal{X} \to \mathcal{X}$ is a self-mapping satisfying the following conditions:*

(i) for all $\widetilde{x}, \widetilde{y} \in \mathcal{X}$

$$\begin{aligned}\widetilde{\alpha}(\widetilde{x}, \widetilde{y}) \cdot \widetilde{d}\left(t^2\widetilde{x}, t^2\widetilde{y}\right) \leq\ &\widetilde{\alpha}_1 \cdot \widetilde{d}(\widetilde{x}, \widetilde{y}) + \widetilde{\alpha}_2 \cdot \widetilde{d}(t\widetilde{x}, t\widetilde{y}) + \widetilde{\alpha}_3 \cdot \widetilde{d}(\widetilde{x}, t\widetilde{x}) + \widetilde{\alpha}_4 \cdot \widetilde{d}\left(t\widetilde{x}, t^2\widetilde{x}\right)\\ &+ \widetilde{\alpha}_5 \cdot \widetilde{d}(\widetilde{y}, t\widetilde{y}) + \widetilde{\alpha}_6 \cdot \widetilde{d}\left(t\widetilde{y}, t^2\widetilde{y}\right)\end{aligned} \tag{28}$$

where $0 \leq \widetilde{\alpha}_i < 1), i = \overline{1,6}$ *such that* $\sum_{i=1}^{6} \widetilde{\alpha}_i < 1$;

(ii) t *is* $\widetilde{\alpha}-$*admissible;*

(iii) *There exists* $\widetilde{x}_0 \in \mathcal{X}$ *such that* $\widetilde{\alpha}(\widetilde{x}_0, t\widetilde{x}_0) \geq 1$;

(iv) t *is orbitally continuous on* $\left(\mathcal{X}, \widetilde{d}\right)$.

Then, t *has a fixed point in* $\mathcal{X}$. *Further, if* t *is* $\widetilde{\alpha}^*$-*admissible, then* t *has a unique fixed point* $\widetilde{z} \in \mathcal{X}$. *Moreover, for any* $\widetilde{x}_0 \in \mathcal{X}$, *if* $\widetilde{x}_{n+1} = t^{n+1}\widetilde{x}_0 \neq t^n\widetilde{x}_0$ *for all* $n \in \mathbb{N} \cup \{0\}$, *then* $\lim_{n \to +\infty} t^n \widetilde{x}_0 = \widetilde{z}$.

Corollary 3. *Let* t *be a continuous mapping on a complete metric space* $\left(\mathcal{X}, \widetilde{d}\right)$ *into itself. If there exists* $k \in [0, 1)$ *satisfying the following inequality*

$$\widetilde{d}\left(t^2\widetilde{x}, t^2\widetilde{y}\right) \leq k \cdot \max\left\{\widetilde{d}(\widetilde{x}, \widetilde{y}), \widetilde{d}(t\widetilde{x}, t\widetilde{y}), \widetilde{d}(\widetilde{x}, t\widetilde{x}), \widetilde{d}\left(t\widetilde{x}, t^2\widetilde{x}\right), \widetilde{d}(\widetilde{y}, t\widetilde{y}), \widetilde{d}\left(t\widetilde{y}, t^2\widetilde{y}\right)\right\} \tag{29}$$

for all $\widetilde{x}, \widetilde{y} \in \mathcal{X}$, *then* t *has a unique fixed point in* $\mathcal{X}$.

The immediate consequences of some of our obtained results in this paper are new contractive conditions that generalize and complement results from [28].

Corollary 4. *Let* $\left(\mathcal{X}, \widetilde{d}\right)$ *be a complete metric space and* t *be a self-mapping satisfying* $F-$*contraction type (7) where* $\widetilde{\tau}_i > 0, i = \overline{1,3}$ *such that for all* $\widetilde{x}, \widetilde{y} \in \mathcal{X}$ *with* $\widetilde{d}(t\widetilde{x}, t\widetilde{y}) > 0$ *the following inequalities holds true:*

$$\widetilde{\tau}_1 + \exp\left(\widetilde{d}(t\widetilde{x}, t\widetilde{y})\right) \leq \exp\left(\widetilde{d}(\widetilde{x}, \widetilde{y})\right) \tag{30}$$

$$\widetilde{\tau}_2 - \frac{1}{\widetilde{d}(t\widetilde{x}, t\widetilde{y})} \leq -\frac{1}{\widetilde{d}(\widetilde{x}, \widetilde{y})} \tag{31}$$

$$\widetilde{\tau}_3 + \exp\left(\widetilde{d}(t\widetilde{x}, t\widetilde{y})\right) \cdot \ln\left(\widetilde{d}(t\widetilde{x}, t\widetilde{y})\right) \leq \exp\left(\widetilde{d}(\widetilde{x}, \widetilde{y})\right) \cdot \ln\left(\widetilde{d}(\widetilde{x}, \widetilde{y})\right). \tag{32}$$

Then, t *has a unique fixed point* $\widetilde{x}^* \in \mathcal{X}$, *and for each* $\widetilde{x} \in \mathcal{X}$, *the sequence* $\{t^n\widetilde{x}\}_{n \in \mathbb{N}}$ *converges to* $\widetilde{x}^*$.

Proof. First, put in Theorem 5 $\mathcal{N}(\widetilde{x}, \widetilde{y}) = \widetilde{d}(\widetilde{x}, \widetilde{y}), \widetilde{F}(\widetilde{r}) = \exp(\widetilde{r})$, $\widetilde{F}(\widetilde{r}) = -\frac{1}{\widetilde{r}}, \widetilde{F}(\widetilde{r}) = \exp(\widetilde{r}) \cdot \ln(\widetilde{r})$, respectively. Since every one of the functions $\widetilde{r} \mapsto \widetilde{F}(\widetilde{r})$ is strictly increasing on $(0, +\infty)$, the result follows by Theorem 5. □

3. Conclusions

This paper considered the results on fixed-point theorems concerning $F-$contraction as presented in the papers mentioned in the abstract. Some significant improvements have been presented, since our approach is based only on the strictly increasing function $\widetilde{F}$ introduced by Wardowski (2012) instead of all three properties presented by the same author that are still largely used. Even though we have proven that only this one property (F1) is sufficient in the case of multivalued mappings, our research raises some new questions:

1. Since $\widetilde{d^p}$ is a continuous b-metric for each metric $\widetilde{d}$ and $p > 1$, it is natural to ask whether Theorem 8 ([26], Theorem 2.) is true if $\widetilde{d^p}$ is replaced by an arbitrary b-metric $\widetilde{d}_b$? For some details see [29];

2. The same question can be asked for Lemma 2 ([26], Lemma 1), that is, is the sequence $\widetilde{d}_b(\widetilde{x}_n, \widetilde{x}_{n+1})$ strictly decreasing, where $\widetilde{d}_b$ is an arbitrary b-metric?;

3. Considering that each strictly increasing function $\widetilde{F} : (0, +\infty) \to (-\infty, +\infty)$ is continuous almost everywhere (also see [30]), it is natural to ask what is the relationship between all three properties (F1), (F2), and (F3), i.e., (F1), (F2’) and (F3’)?

It is obvious that the results will open new perspectives (for example [31–33]) and topics for researchers and therefore, this paper will be useful for new studies related to fixed-point theory.

Author Contributions: Conceptualization, S.R. and J.V.; methodology, N.F., M.T. and S.R.; software, N.K. and N.F.; validation, J.V., N.F. and S.R.; formal analysis, N.F. and J.V.; investigation, S.R. and J.V.; data curation, N.K. and M.T.; writing—original draft preparation, M.T., N.K. and N.F.; writing—review and editing, J.V., S.R. and N.K.; visualization, M.T.; supervision, J.V.; All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Acknowledgments: This work is supported by Faculty of Sciences and Mathematics, University of Priština in Kosovska Mitrovica, Serbia, project no. IJ-0203.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Banach, S. Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales. *Fund. Math.* **1922**, *3*, 133–181. [CrossRef]
2. Ćirić, L. A generalization of Banach's contraction principle. *Proc. Am. Math. Soc.* **1974**, *45*, 267–273. [CrossRef]
3. Istratescu, V.I. Some fixed point theorems for convex contraction mappings and convex nonexpansive mappings. *Lib. Math.* **1981**, *1*, 151–164.
4. Alghamdi, M.A.; Alnafei, S.H.; Radenović, S.; Shahzad, N. Fixed point theorems for convex contraction mappings on cone metric spaces. *Math. Comput. Model.* **2011**, *54*, 2020–2026. [CrossRef]
5. Filipović, M.G.; Kukić, K.; Mitrović, Z.D.; Radenović, S. Review of the convex contractions of Istratescu's type in various generalized metric spaces. *Adv. Theory Nonlinear Anal. Its Appl.* **2020**, *3*, 121–131.
6. Miculescu, R.; Mihail, A. A generalization of Istratescu's fixed point theorem for convex contractions. *arXiv* **2015**, arXiv:1512.05490v1.
7. Mitrović, Z.D.; Aydi, H.; Mlaiki, N.; Filipović, M.G.; Kukić, K.; Radenović, S.; De la Sen, M. Some new observations and results for convex contractions of Istratescu's type. *Symmetry* **2019**, *11*, 1457. [CrossRef]
8. Kumam, P.; Dung, N.V.; Sitthithakemgkiet K. A generalization of Ćirić fixed point theorem. *Filomat* **2015**, *29*, 1549–1556. [CrossRef]
9. Wardowski, D. Fixed points of a new type of contractive mappings in complete metric spaces. *Fixed Point Theory Appl.* **2012**, *2012*, 94. [CrossRef]
10. Aljančić, S. *Uvod u Realnu i Funkcionalnu Analizu*; Naučna knjiga: Beograd, Serbian, 1969.
11. Chandok, S.; Radenović, S. Existence of solution for orthogonal F-contraction mappings via Picard-Jungck sequences. *J. Anal.* **2021**, 1–14. [CrossRef]
12. Mirkov, N.; Radenović, S.; Radojević, S. Some new observations for F-contractions in vector-valued metric spaces of Perov's type. *Axioms* **2021**, *10*, 127. [CrossRef]
13. Miculescu, R.; Mihail, A. New fixed point theorems for set-valued contractions in b-metric spacers. *J. Fixed Point Theory Appl.* **2017**, *19*, 2153–2163. [CrossRef]
14. Piri, H.; Kumam, P. Fixed point theorems for generalized F-Suzuki-contraction mappings in complete b-metric spaces. *Fixed Point Theory Appl.* **2016**, *2016*, 90. [CrossRef]
15. Qawaqneh, H.; Noorani, M.S.; Shatanawi, W. Fixed point results for Geraghty type generalized F-contraction for weak α-admissible mappings in metric-like spaces. *Eur. J. Pure Appl. Math.* **2018**, *11*, 702–716. [CrossRef]
16. Radenović, S.; Mirkov, N.; Paunović, L.R. Some new results on F-contractions in 0-complete partial metric spaces and 0-complete metric-like spaces. *Fractal Fract.* **2021**, *5*, 34. [CrossRef]
17. Rudin, W. *Principles of Mathematical Analysis*; McGRAW-HILL Book Company: New York, NY, USA; San Franciscko, CA, USA; Toronto, ON, Canada; London, UK, 1964.
18. Vujaković, J.; Mitrović, S.; Mitrović, Z.D.; Radenović, S. On F-contractions for weak α-admissible mappings in metric-like spaces. *Mathematics* **2020**, *8*, 1629. [CrossRef]
19. Vujaković, J.; Radenović, S. On some F-contraction of Piri-Kumam-Dung-type mappings in metric spaces. *Mil. Tech. Cour.* **2020**, *68*, 697–714. [CrossRef]
20. Wardowski, D.; Dung, N.V. Fixed points of F-weak contractions on complete metric spaces. *Demonstr. Math.* **2014**, *47*, 146–155. [CrossRef]
21. Dung, N.V.; Hang, V.I. A fixed point theorem for generalized F-contractions on complete metric spaces. *Vietnam. J. Math.* **2015**, *43*, 743–753. [CrossRef]
22. Piri, H.; Kumam, P. Some fixed point theorem concerning F-contraction in complete metric spaces. *Fixed Point Theory Appl.* **2014**, *2014*, 210. [CrossRef]
23. Karapinar, E.; Fulga, A.; Petrusel, A. On Istratescu Type Contractions in b-Metric Spaces. *Mathematics* **2020**, *8*, 388. [CrossRef]

24. Karapinar, E.; Kumam, P.; Salimi, P. On α-ψ-Meir-Keeler contractive mappings. *Fixed Point Theory Appl.* **2013**, *2013*, 94. [CrossRef]
25. Samet, B.; Vetro, C.; Vetro, P. Fixed point theorems for α-ψ-contractive type mappings. *Nonlinear Anal.* **2012**, *75*, 2154–2165. [CrossRef]
26. Singh, Y.M.; Khan, M.S.; Kang, S.M. F-convex contraction via admissible mapping and related fixed point theorems with an application. *Mathematics* **2018**, *6*, 105; Correction in *Mathematics* **2019**, *7*, 337. [CrossRef]
27. Radenović, S.; Vetro, F.; Vujaković, J. An alternative and easy approach to fixed point results via simulation functions. *Demonstr. Math.* **2017**, *5*, 224–231. [CrossRef]
28. Ćirić, L.J. *Some Recent Results in Metrical Fixed Point Theory*; University of Belgrade: Beograd, Serbia, 2003.
29. Popescu, O.; Stan, G. Some fixed point theorems for quadratic quasicontractive mappings. *Symmetry* **2019**, *11*, 1329. [CrossRef]
30. Popescu, O.; Stan, G. Two fixed point theorems concerning F-contraction in complete metric spaces. *Symmetry* **2021**, *12*, 58. [CrossRef]
31. Asim, M.; Uddin, I.; Imdad, M. Fixed point results in M í-metric spaces with an application.*J. Inequal. Appl.* **2019**, *2019*, 280. [CrossRef]
32. Uddin, I.; Perveen, A.; Isik, H.; Bhardwaj, R. A Solution of Fredholm Integral Inclusions via Suzuki-Type Fuzzy Contractions. *Math. Probl. Eng.* **2021**, *2021*, 6579405. [CrossRef]
33. Perveen, A.; Uddin, I.; Imdad, M. Generalized contraction principle under relatively weaker contraction in partial metric spaces. *Adv. Differ. Equ.* **2019**, *2019*, 88. [CrossRef]