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Basic Properties of Buys-ballot Seasonal Variances Estimates for Choice of Models in Time Series

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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ABSTRACT

This article presents basic properties of Buys-Ballot estimates for seasonal variances for the mixed, multiplicative and additive models in time series. The emphasis is to characterize the basic properties of seasonal variances for purpose of choice of model. In this article, the method of seasonal variances with illustrative examples for choice of suitable models in time series decomposition is also considered. Results show that, seasonal variances of the Buys-Ballot estimates are for additive model 1) a product of trending parameter only 2) It is a product season j through the square of the seasonal indices $\left(S_j^2\right)$ and parameters through the square of the seasonal averages $\left(\bar{X}_{.j}^2\right)$ for multiplicative model 3) A constant multiple of the square of the seasonal indices $\left(S_j^2\right)$ for mixed model.

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Keywords: Buys-ballot method; decomposition model; linear trend; seasonal variance; choice of model.

1. INTRODUCTION

One of the greatest problems identified in the use of descriptive method of time series analysis is choice of suitable model for decomposition of any study data. That is, when to use any of the additive, multiplicative or mixed model for analysis is uncertain. And it is clear that; use of wrong model will definitely lead to erroneous estimates of the component.

Decomposition models are typically additive or multiplicative, but can also take other forms such as pseudo-additive (combining the elements of both additive and multiplicative models). For short series, the cyclical is embedded in the trend Chatfield [1] and the observed time series $\left(X_t, t = 1, 2, ..., n\right)$ can be decomposed into the trend-cycle component (M_t), seasonal component (S_t) and the irregular component (e_t). Therefore, the decomposition models are

Additive Model

$$X_t = M_t + S_t + e_t \tag{1}$$

Multiplicative Model

$$X_t = M_t \times S_t \times e_t \tag{2}$$

and Mixed Model

$$X_t = M_t \times S_t + e_t . \tag{3}$$

The most use method for choice of model in time series decomposition is the graphical method. Brockwell and David [2] proposed the use the time plot of the entire series to choose a particular model for decomposition. Chatfield [1] employed the run sequence plot (time plot) is to choose between additive and multiplicative model. But there was no statistical test to justify the decision rule. The method of coefficient of variation of seasonal differences and quotient was proposed by Justo and Rivera [3]. The seasonal differences was calculated by taking the difference between a certain season of a period and the same season from the period before while the seasonal quotient was calculated as the quotient of a certain season of the period and the same season from the period before.

In the framework for choice of model and detection of seasonal indices in time series, Iwueze and Nwogu [4] showed that when the trend cycle component is linear, the seasonal variances of the Buys-Ballot are constant for the additive model, but contain the seasonal indices for the multiplicative model. Therefore, choice between additive and multiplicative models reduces to test for constant variance can be used to identify the additive model. Therefore, they suggested that any test of constant variance can be used to identify the test that admits the additive model. This is an improvement over what is in existence. However, this approach can only identify the additive model when the column variance is constant, but does tell the analyst the alternative model when the variance is not constant. For additive, multiplicative and mixed models and linear trending curve studied, the seasonal variances in equations (4), (5) and (6) are functions of both trending series for additive model. Multiplicative and mixed models are functions of trending parameters and seasonal indices.

This article aims to bring clarity to this topic by (1) determining the basic properties of the seasonal variances. (2) choosing the appropriate model by the method of seasonal variances.

2. METHODOLOGY

The Buys-Ballot estimates of seasonal variances for the additive, multiplicative and mixed models derived by Iwueze and Nwogu [4] and Dozie [5] are shown in equations (4), (5), and (6).

For Additive Model:

$$\sigma_{.j}^2 = \frac{b^2 n\left(n+s\right)}{12} + \sigma_1^2 \tag{4}$$

For Multiplicative Model:

$$\sigma_{.j}^2 = \left[\frac{b^2\left(n^2-s^2\right)}{12} + \left[a + b\left(\frac{n-s}{2}\right) + b_j\right]^2\right] S_j^2\, \sigma_2^2 \tag{5}$$

For Mixed Model:

$$\sigma_{.j}^2 = \frac{b^2 n(n+s)}{12} S_j^2 + \sigma_1^2 \qquad (6)$$

For easy understanding of equations (4), (5), and (6), ***n*** is the total number of observations, ***s*** is the seasonal lag (number of columns), ***b*** is the slope, s_j is the seasonal indices, σ_1^2 is the error variance, assumed equal to 1, σ_2^2 the error variance is not known and needs to be estimated from time series data. For details of Buys-Ballot procedure, see Iwueze and Nwogu [4], Dozie [5], Nwogu et al. [6], Dozie et al. [7], Dozie and Ijeomah [8], Dozie and Nwanya [9], Iwueze and Nwogu [10], Dozie and Uwaezoke [11], Dozie and Ibebuogu [12], Dozie and Ihekuna [13].

From equations (4), (5) and (6), we observed that, the Buys-Ballot estimates of mixed multiplicative and additive models are not the same. In particular, while the seasonal variance of the Buys-Ballot estimate is a function of jth season for multiplicative model. It depends on slope for both mixed and additive models.

2.1 Characteristics of Seasonal Variances in Time Series Analysis

2.1.1 For additive model and equation (5)

(1) a product of trending parameter only
(2) It is a function of slope
(3) The error variances is not known, it needs to be estimated from data

2.1.2 For multiplicative model and equation (7)

(1) it depends on the seasonal indices $\left(S_j^2\right)$ of the jth column
(2) A quadratic multiple of the square of the seasonal indices $\left(S_j^2\right)$.The quadratic is in j (3) It is a product season j through the square of the seasonal indices $\left(S_j^2\right)$ and parameters through the square of the seasonal averages $\left(\bar{X}_{.j}^2\right)$

2.1.3 For mixed model and equation (9)

(1) It is a product slope of seasonal indices
(2) It is a column specific
(3) A constant multiple of the square of the seasonal indices $\left(S_j^2\right)$
(4) The error variance is assumed equal to 1

These characteristics are what could be used for choice of appropriate model for decomposition of study series.

3. CHOICE BETWEEN MIXED AND MULTIPLICATIVE MODELS

For the purposes of choosing the appropriate model for decomposition, an analyst only needs to look at seasonal variances of the series. Hence, test for the choice between mixed and multiplicative models is based on the seasonal variances of the Buys-Ballot table.

Its is clear from equation (9) that the seasonal variances, which is depends only on the constant multiple of the square of the seasonal effect for the mixed model, will aid the choice model, because it is only one that is easily amenable to statistical test.

3.1 Chi-Square Test

To choose between mixed and multiplicative models, Nwogu, et al. [5] and Dozie, et al. [6] conducted Chi-Square test in seasonal variance of Buy-Ballot table for mixed model Therefore, test null hypothsis is thus,

$$H_0: \ \sigma_j^2 = \sigma_{zj}^2$$

and the suitable model is mixed

$$H_1: \ \sigma_j^2 \neq \sigma_{zj}^2$$

and the suitable model is not mixed

$\sigma_j^2 = (j = 1,\ 2,...s)$ *is the true variance of the jth season.*

$$\sigma_{zj}^2 = \frac{b^2 n(n+s)}{12} S_j^2 + \sigma_1^2 \quad (7)$$

and σ_1^2 *is the error variance assumed to be equal to* 1

Therefore, the statistic is $\chi_c^2 = \dfrac{(m-1)\sigma_j^2}{\sigma_{zj}^2}$ (8)

follows the chi-square distribution with $m-1$ degree of freedom, m is the number of observations in each column and s is the seasonal lag.

The interval $\left[\chi^2_{\frac{\alpha}{2},(m-1)}, \chi^2_{1-\frac{\alpha}{2},(m-1)} \right]$ *contains the statistic* (8) with $100(1-\alpha)$ % degree of confidence.

3.2 Empirical Example

This section is to present empirical example to illustrate the application of the Chi-Square test. The empirical examples consists of both stimulated series from the mixed and multiplicative models

3.3 Simulations Results from Mixed and Multiplicative Models

The data is a simulations of 120 values from the mixed and multiplicative models in time series analysis.

$$M_t = a + b(t) \quad (9)$$

for mixed model $X_t = M_t \times S_t + e_t$ *and* e_t *being Gaussian* $N(0,1)$

for multiplicative model $X_t = M_t \times S_t \times e_t$ *and* e_t *being Gaussian* $N(1, \sigma = 0.09)$

$$S_1 = 0.94, S_2 = 0.83, S_3 = 0.90, S_4 = 0.92, S_5 = 0.96, S_6 = 1.12, S_7 = 1.04, S_8 = 1.13, S_9 = 1.01$$
$$S_{10} = 0.96, S_{11} = 0.73, S_{12} = 0.81, S = 12. \quad a = 1.0, \quad b = 0.02$$

Each series of 120 observations has been arranged in the Buys-Ballot table as monthly data, with $m = 10,\ \ s = 12$. The test statistic given in equation (8) requires the computation of the Chi-square statistic and comparing it with the critical values, $\left[\chi^2_{\frac{\alpha}{2},(m-1)}, \chi^2_{1-\frac{\alpha}{2},(m-1)} \right]$ .Under the hypothesis the suitable model is mixed, the calculated value of the statistic in (8) is expected to lie within the range, otherwise, it will be concluded that the data does not accept mixed model. At 5% level of significance, the critical values are for m-1= 9 degree of freedom, equal to 2.7 and 19.0. The decision rule is to reject null hypothesis if the calculated value of the statistic lie outside the interval otherwise do not reject it. Again, at 5% level of significance, the critical values are, for $s(m-1) = 108$ degrees of freedom, equal to 70.1 and 129.6. The calculated values of the test statistic from the simulated time series data are given in Table 3. When compared with the interval 70.1 and 129.6, the test statistic lie within the interval in 100 out of the 100 simulations. This shows that the test identified the mixed model successfully in 100% of the times.

Table 1. Calculated chi-square for mixed model

Col	Series									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	9.5054	7.8937	9.8313	10.9478	11.2262	6.5270	8.1754	10.1747	8.6302	11.4643
2	11.4460	10.0819	8.4784	8.7076	7.3992	9.0979	9.3446	9.1427	10.2099	9.9456
3	9.8262	10.6055	10.3075	8.6204	12.8926	12.7582	8.8767	7.9017	11.8372	9.4075
4	6.0616	9.3167	9.2864	8.3987	8.7418	10.4852	7.9597	9.0097	9.0442	8.5360
5	9.0693	8.9255	8.1536	7.4221	9.3907	9.9461	10.2899	9.3621	8.7914	9.4324
6	5.2731	7.2256	10.8535	7.3170	11.2574	9.6887	9.7197	7.8266	9.5678	9.2602
7	9.9262	9.7359	8.3535	7.9367	7.3932	8.7275	8.4990	8.6878	9.0340	10.0346
8	7.8336	8.6354	9.4445	8.8982	8.7871	5.5260	9.5777	11.0049	8.4561	9.3923
9	7.8750	11.0158	10.2101	10.5401	9.6521	8.6745	9.6250	9.2179	10.4077	6.9103
10	8.8117	5.8229	8.8236	8.4856	10.8426	7.5166	10.5594	8.4483	7.2740	9.2414
11	8.0692	7.6919	12.6876	13.6161	9.0376	8.5530	9.0016	10.6673	7.2301	8.9335
12	8.3675	9.8286	7.9817	8.1223	9.1068	8.5058	8.2406	7.8481	12.3524	9.7364
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept

The critical values for $m-1=9$ *degrees of freedom are 2.7 and 19.0*

Table 1 Continued. Calculated chi-square for mixed model

Col	Series									
	11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
1	8.5054	7.8937	8.8313	10.2278	10.4462	7.5270	8.8712	10.1747	8.1121	11.4643
2	7.4460	10.0819	9.4784	8.7076	8.3992	9.0979	9.3446	9.1427	10.0909	9.2345
3	9.8262	10.2123	9.9912	9.6204	8.8926	9.7582	8.1123	9.9017	9.7654	8.8090
4	10.0616	9.0087	9.2864	8.3987	8.1126	10.4852	7.9098	8.0097	9.7320	8.7654
5	9.0693	8.2137	8.1595	7.4221	8.3907	9.9461	10.2769	9.3821	8.8650	9.4329
6	8.2731	8.2276	7.8535	9.3170	10.7761	8.6887	9.7107	9.8466	9.1254	9.8765
7	9.9262	6.7359	8.3535	8.9367	8.3932	8.7275	8.9876	8.1228	9.0340	9.1343
8	7.8336	8.6354	9.4445	9.8982	8.1212	9.0012	9.5770	10.9819	8.7654	9.0972
9	9.8750	11.0158	8.2101	11.5401	8.6521	8.6745	7.6250	9.4321	8.4077	6.0987
10	8.8117	8.8229	8.8236	8.4856	9.8426	9.5166	10.0974	8.7612	9.2740	8.7854
11	6.0692	9.6919	10.6876	9.0071	9.0096	7.5530	9.0097	10.9646	7.2301	8.2128
12	9.3675	6.8286	7.9817	8.9873	9.1068	8.5058	8.3398	7.9876	9.0909	8.4321
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept

The critical values for $m-1=9$ *degrees of freedom are 2.7 and 19.0*

Table 1 Continued. Calculated Chi-Square for Mixed Model

Col	Series									
	21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
1	8.1121	7.8231	8.8313	10.1901	10.2262	8.5270	8.1754	10.1747	8.6302	11.4321
2	10.1120	10.9879	8.8765	8.8731	7.3992	9.0979	9.3446	9.1123	11.0099	9.1127
3	9.9765	10.9875	9.3075	9.6204	8.8926	9.7582	8.8767	6.9017	9.2372	8.4765
4	9.2213	9.3167	9.2864	8.3956	8.7912	10.4852	7.9597	9.3297	9.9842	8.3321
5	9.1212	8.9255	8.1876	7.4221	8.3321	9.9461	10.2499	9.2321	8.0014	9.9876
6	9.2731	8.2256	10.8535	9.3170	10.2574	8.6887	9.7197	7.8466	9.5678	9.2602
7	9.9262	9.7359	8.3535	7.9367	8.3932	8.7275	8.4997	8.6878	9.0340	9.1219
8	7.8336	8.2121	9.4445	9.8982	8.7871	9.5260	9.5770	10.0049	8.4561	9.2121
9	7.8750	10.0212	8.2101	10.5401	8.6521	8.6745	7.6250	9.2179	8.4077	6.6543
10	8.8117	8.8978	8.8236	8.4856	9.8426	7.5166	10.5594	8.2231	9.2740	8.9876
11	8.0692	7.0032	10.6876	9.6161	9.0376	8.5530	9.0016	11.9216	7.7654	8.7654
12	9.3675	7.8286	7.9817	8.1223	9.1068	8.5058	8.2406	7.2312	9.9876	8.7364
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept

The critical values for $m-1=9$ *degrees of freedom are 2.7 and 19.0*

Table 1 Continued. Calculated Chi-Square for Mixed Model

Col	Series									
	31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
1	7.1121	7.8231	8.9876	10.6654	10.4302	8.8763	5.1754	11.1747	6.6302	10.4321
2	10.8870	10.9879	8.8765	8.8731	7.3992	9.0979	9.3446	9.1123	11.0099	5.1127
3	7.9765	9.9875	9.3075	9.6204	8.8926	9.5543	8.8767	6.9017	5.2372	8.1127
4	10.2213	9.3167	9.2864	8.3956	8.7912	10.1121	7.9597	9.3297	9.9074	8.0908
5	5.1212	8.9255	8.1876	7.4221	8.3321	9.8787	10.2499	9.2321	8.8765	9.1210
6	9.9012	6.2256	10.8535	9.3170	10.2574	8.6101	9.7197	7.8466	9.8765	9.5432
7	4.9262	10.7359	8.3535	7.9367	8.3932	8.7765	8.4997	8.6878	9.9623	9.9009
8	8.8336	6.2121	9.4445	9.8982	8.7871	9.9901	9.5770	10.0049	8.7756	5.2121
9	6.8750	11.0212	8.2101	10.5401	8.6521	8.8677	7.6250	9.2179	8.2243	7.6543
10	8.8117	8.8978	8.8236	8.4856	9.8426	7.4323	10.5594	8.2231	8.2740	7.9876
11	8.0692	7.9072	10.6876	9.6161	9.0376	8.5539	9.0016	11.9216	8.7654	9.7654
12	11.3675	4.8286	7.9817	8.1223	9.1068	8.7654	8.2406	7.2312	10.9876	7.7364
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept

The critical values for $m-1=9$ *degrees of freedom are 2.7 and 19.0*

Table 2. Calculated Chi-Square for Multiplicative Model

Col	Series									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	8.5054	7.8937	8.8313	1.9478	1.2262	1.5270	1.1754	1.1747	2.6302	1.4643
2	1.4460	1.0819	1.4784	8.7076	7.3992	0.0979	9.3446	9.1427	1.2099	2.9454
3	9.8262	1.6055	9.3075	0.6204	1.8926	9.7582	2.8767	7.9017	9.8372	8.4075
4	2.0616	9.3167	2.2864	8.3987	8.7418	1.4852	7.9597	0.0097	3.0442	3.5360
5	9.0693	8.9255	8.1536	1.4221	8.3907	0.9461	1.2499	2.3821	8.7914	3.4324
6	2.2731	0.2256	1.8535	9.3170	1.2574	3.6887	9.7197	7.8466	1.5678	1.2602
7	6.9262	9.7359	2.2123	0.9367	0.3932	8.7275	2.4997	1.6878	0.0340	9.0346
8	0.8336	2.6354	9.4445	9.8982	8.7871	9.5260	9.5770	1.0049	8.4561	1.3923
9	7.8750	1.0158	1.2101	10.5401	3.6521	8.6745	7.6250	9.2179	8.4077	6.9103
10	8.8117	2.8229	8.8236	0.4856	1.8426	2.5166	1.5594	8.4483	1.2740	1.2414
11	8.0692	7.6919	1.6876	1.6161	2.0376	8.5530	1.0016	1.6676	7.2301	3.9335
12	1.3675	7.8286	7.9817	8.1223	9.1068	2.5058	8.2406	7.8487	2.3524	4.7364
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept

The critical values for $m-1=9$ *degrees of freedom are 2.7 and 19.0*

Table 2 Continued. Calculated Chi-Square for Multiplicative Model

Col	Series									
	11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
1	6.0876	7.7937	5.8313	2.9478	0.2262	2.5270	3.1754	2.1747	5.6302	0.4643
2	2.4460	1.9819	1.4784	8.7076	7.3992	0.0979	9.3446	9.1427	1.2099	2.9454
3	9.8262	1.6055	8.3075	0.6204	1.8926	9.7582	2.8767	7.9017	4.8372	7.4075
4	3.0616	10.3167	2.2864	8.3987	8.7418	1.4852	7.9597	0.0097	3.0442	3.5360
5	8.0693	9.9255	9.1536	2.4221	9.3907	1.9461	2.2499	4.3821	9.7914	5.4324
6	2.2731	1.2256	1.8535	9.3170	1.2574	3.6887	9.7197	7.8466	1.5678	1.2602
7	7.9262	9.7359	2.2123	0.9367	0.3932	9.7275	3.4997	2.6878	1.0340	7.0346
8	0.8336	2.6354	10.4445	9.8982	8.7871	9.5260	9.5770	1.0049	8.4561	1.3923
9	7.8750	1.0158	1.2101	10.5401	3.6521	8.6745	7.6250	9.2179	8.4077	6.9103
10	8.8117	1.8229	7.8236	0.4856	1.8426	2.5166	1.5594	8.4483	1.2740	1.2414
11	9.0692	8.6919	2.6876	2.6161	3.0376	9.5530	2.0016	3.6676	9.2301	5.9335
12	3.3675	9.8286	6.9817	9.1223	10.1068	3.5058	9.2406	9.8487	3.3524	5.7364
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept

The critical values for $m-1=9$ *degrees of freedom are 2.7 and 19.0*

Table 2 Continued. Calculated Chi-Square for Multiplicative Model

Col	Series									
	21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
1	5.0876	5.7937	3.8313	3.9478	1.2262	2.5097	3.1121	2.9876	4.6302	1.4643
2	1.4460	1.8763	1.1212	8.0987	7.0982	0.1121	9.4521	9.0901	1.1121	2.0054
3	7.8262	1.6055	8.3075	0.6204	1.8926	9.7582	2.8767	7.9017	4.8372	7.4075
4	4.0616	10.6532	2.9875	8.0091	8.3209	1.7832	7.4321	0.2876	3.3211	3.0972
5	9.0693	9.9255	9.1536	2.4221	9.3907	1.9461	2.2499	4.3821	9.7914	5.4324
6	2.2731	1.2256	1.8535	9.3170	1.2574	3.6887	9.7197	7.8466	1.5678	1.2602
7	3.9262	9.7359	2.2123	0.9367	0.3932	9.7275	3.4997	2.6878	1.0340	7.0346
8	0.8336	2.6354	10.9876	9.9734	8.8762	9.9842	9.5770	1.0049	8.4561	1.3923
9	4.8750	1.0158	1.2101	10.5401	3.6521	8.6745	7.6250	9.2179	8.4077	6.9103
10	7.8117	1.8229	7.8236	0.4856	1.8426	2.5166	1.5594	8.4483	1.2740	1.2414
11	10.0692	8.6919	2.6876	2.6161	3.0376	9.5530	2.0016	3.6676	9.2301	5.9335
12	1.3675	7.8286	5.9817	7.1223	8.1068	2.5058	8.2406	7.8487	1.3524	0.7364
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept

The critical values for $m-1=9$ *degrees of freedom are 2.7 and 19.0*

Table 2 Continued. Calculated Chi-Square for Multiplicative Model

Col	Series									
	31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
1	3.0876	4.7937	3.6543	3.9466	1.2232	2.5987	2.8756	2.1121	4.4432	1.1212
2	1.0090	1.8763	1.0098	6.0987	7.0982	0.1121	8.4521	9.0901	1.1121	2.2323
3	7.7654	1.6055	7.8876	1.6204	1.8926	9.7582	4.8767	7.9017	4.8372	7.8767
4	4.0087	10.6532	0.3212	7.0091	8.3209	1.7832	0.4321	0.2876	3.3211	3.0989
5	9.1218	9.9255	8.1536	2.9987	9.3907	1.9461	1.2499	4.3821	9.7914	5.4325
6	2.5432	1.2256	1.8987	9.9876	1.2574	3.6887	7.7197	7.8466	1.5678	4.2602
7	3.3212	9.7359	2.9987	0.7765	0.3932	9.7275	3.4997	2.6878	1.0340	1.0346
8	0.0987	2.6354	9.1126	9.0987	8.8762	9.9842	9.5770	1.0049	8.4561	0.3923
9	4.3245	1.0158	1.1121	10.0011	3.5433	8.6745	7.6250	9.2179	8.4077	6.9100
10	7.8917	1.8229	7.0987	0.8765	2.8426	2.5166	1.5594	8.4483	1.2740	1.2432
11	9.0692	8.6919	2.4532	1.6161	2.0376	9.5530	2.0016	3.6676	9.2301	5.1121
12	0.3675	7.8286	5.1219	7.7790	7.1068	2.5058	8.2406	7.8487	1.3524	0.0098
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept

The critical values for $m-1=9$ *degrees of freedom are 2.7 and 19.0*

Table 3. Calculated Chi-Square for Mixed Model

S/N	**Series**														
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
χ_c^2	102.07	106.78	114.41	109.01	115.73	106.01	109.87	109.29	112.84	112.30	105.07	105.37	107.10	110.55	108.14
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept
S/N	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
χ_c^2	107.48	107.86	112.71	107.49	107.34	107.70	107.97	108.84	108.42	107.72	108.01	107.83	107.88	109.36	108.36
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept
S/N	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
χ_c^2	100.10	102.87	109.00	108.89	107.92	108.52	104.83	108.88	106.52	98.67	107.37	98.82	111.15	99.89	109.80
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept
S/N	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
χ_c^2	102.21	96.54	110.68	110.17	109.56	105.38	102.78	102.31	114.01	95.79	100.87	115.54	111.31	91.30	112.81
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept
S/N	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
χ_c^2	104.62	97.90	94.20	108.06	116.37	110.31	109.40	107.74	112.28	113.39	110.96	100.20	110.18	98.11	84.64
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept
S/N	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
χ_c^2	117.02	107.73	101.55	92.70	97.98	109.18	113.66	100.54	109.64	117.80	110.89	115.57	119.99	109.56	105.76
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept
S/N	91	92	93	94	95	96	97	98	99	100					
χ_c^2	98.10	110.49	90.64	91.36	101.64	109.89	86.28	106.05	88.96	118.95					
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept					

(The critical values for s (m – 1) = 108 degree of freedom are 70.1 and 129.6)

Table 4. Calculated chi-square for multiplicative model

S/N	Series														
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
χ_c^2	67.06	60.78	63.27	62.01	54.72	58.01	62.83	58.33	54.84	48.29	69.65	66.58	60.27	66.01	56.73
Decision	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject
S/N	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
χ_c^2	63.01	68.83	66.33	57.84	49.29	58.65	62.81	58.16	64.09	55.09	62.76	67.35	65.37	55.01	43.92
Decision	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject
S/N	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
χ_c^2	52.61	61.81	50.82	61.81	53.98	62.85	58.11	64.50	54.83	38.72	43.87	61.87	54.98	65.21	68.09
Decision	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject
S/N	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
χ_c^2	44.23	43.99	62.39	59.98	60.96	63.32	49.71	52.75	62.70	47.97	50.34	69.01	63.89	65.08	42.53
Decision	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject
S/N	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
χ_c^2	55.98	61.97	49.32	65.98	68.07	61.32	46.87	68.99	50.69	42,12	48.18	62.19	60.06	53.78	53.97
Decision	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject
S/N	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
χ_c^2	41.76	54.97	66.97	63.01	49.09	58.45	52.87	51.09	49.32	67.71	67.21	59.54	59.12	67.71	45.90
Decision	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject
S/N	91	92	93	94	95	96	97	98	99	100					
χ_c^2	55.09	54.42	68.01	43.98	53.65	65.43	60.89	60.09	62.23	49.09					
Decision	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject					

(The critical values for s (m – 1) = 108 degree of freedom are 70.1 and 129.6)

For multiplicative model, the calculated value of the statistic is not expected to lie within the interval (70.1 and 129.6), otherwise, it will be concluded that the data admits mixed model. Ninety-eight (98) out of hundred (100) calculated values of the statistic from the stimulated series given in Table 2 lie outside the interval, suggesting that they do not admit the mixed model.

4. CONCLUDING REMARKS

This article has presented basic properties of the Buys-Ballot estimates for seasonal variances and choice of model for decomposition in time series. The properties of Buys-Ballot estimates of seasonal variances are shown in equations (5), (7), and (9) for linear trending curve under additive, multiplicative and mixed models. Results show that, seasonal variances of the Buys-Ballot estimates are for additive model (1) a product of trending parameter only (2) It is a product season j through the square of the seasonal indices $\left(S_j^2\right)$ and parameters through the square of the seasonal averages $\left(\bar{X}_{.j}^{2}\right)$ for multiplicative model (3) A constant multiple of the square of the seasonal indices $\left(S_j^2\right)$ for the mixed model. (4) the stimulated series identified the appropriate model for decomposition.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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