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Generalized Fixed Points for Four Self-mappings with the Property OWC in CMS

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Author's contribution

The sole author designed, analyzed, interpreted and prepared the manuscript.

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Abstract

In this research article, we obtained fixed points for Four self – mappings with the property Occasionally Weakly Compatible (OWC) in Cone Metric Spaces (CMS). Our results are generalized, improved some of the results in this article references.

Keywords: Cone metric space; common fixed point; fixed point theorem and occasionally weakly compatible.

1 Introduction

Cone metric space concept introduced by. Huang and Zhang, [1] they replacing the real numbers by an ordered Banach space(B-space) , also proved fixed point results in this CMS, after that my authors have been generalized ,extended and improved the fixed point theorems of Huang and Zhang [1] using with different types of contractive conditions [see, for eg. 3-6, 7-11] . And recently Bhatt and Chandra [2] proved some of the results on OWC mappings in CMS. In this article we generalized and improved and extended the results of Bhatt and Chandra [2].

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2 Preliminaries

We have defined some definitions and which are useful in our main results and these are existing in the references of this article.

Definition 2.1.

Let S be a real B- space and $B \subset S$. And the set "P" is said to be cone iff

(i) $B \neq \{\emptyset\}$ and $B \neq \{0\}$, B is closed;
(ii) $\Rightarrow \alpha x+\beta y \in B$, $\alpha, \beta \geq 0 \in \mathbb{R}$, and $x, y \in B$
(iii) If both x and $-x \in B \Rightarrow x = 0$.

Definition 2.2.

Let B be a cone in a B- space S. And the partial ordering '$\leq$' w. r. t. to B by $x_1 \leq x_2 \Leftrightarrow x_2 - x_1 \in B$. We shall write $x_1 < x_2$ to indicate $x_1 \leq x_2$ but $x_1 \neq x_2$ while $x_1 \ll x_2$ will stand for $x_2 - x_1 \in$ interior of B. And this type of cone B is said to be an order cone.

Definition 2,3.

Let S be a B-space and $B \subset S$ be an order cone. And the order cone B is said to be normal cone if there exists a constant $N > 0$ such that for all $x_1, x_2 \in S$,

$0 \leq x_1 \leq x_2$ implies that $\|x_1\| \leq N \|x_2\|$.

The smallest positive number N satisfies this type of inequality is said to be the normal constant of B.

Definition 2.4.

Let $X_1 \neq \{\emptyset\}$ set of S. And suppose that the mapping $\rho : X_1 \times X_1 \rightarrow S$ satisfies the following properties

(d1). $0 \leq \rho(x_1, y_1)$ for all $x_1, y_1 \in X_1$ and $\rho(x_1, y_1) = 0 \Leftrightarrow x_1 = y_1$;
(d2). $\rho(x_1, y_1) = \rho(y_1, x_1)$ for all $x_1, y_1 \in X_1$;
(d3). $\rho(x_1, y_1) \leq \rho(x_1, z_1) + \rho(z_1, y_1)$ for all $x_1, y_1, z_1 \in X_1$.

Then ρ is said to be a cone metric on X_1 and (X_1, ρ) is said to be a CMS.

Note that the concept of a CMS is more general than the metric space.

Example 2.5

Let $S = \mathbb{R}^2$, $B = \{(x_1, y_1) \in S$ such that : $x_1, y_1 \geq 0\} \subset \mathbb{R}^2$, $X_1 = \mathbb{R}$ and $\rho : X_1 \times X_1 \rightarrow S$ such that $\rho(x_1, y_1) = (|x_1 - y_1|, \alpha |x_1 - y_1|)$, where α is a constant and is ≥ 0. Then (X_1, ρ) is said to be a CMS.

Definition 2.6

Suppose that (X_1, ρ) is a CMS. If the sequence $\{x_n\}$ is said to a convergent sequence

(i) for any $b \gg 0$, $\exists$ a positive integer $N \ni \rho(x_n, x_1) \ll b$ for all $n > N$, for some fixed point x_1 in X_1. And denote $x_n \rightarrow x_1$ as $n \rightarrow \infty$.

And (ii) If the sequence $\{x_n\}$is said to be a Cauchy sequence if for every b in S with $b \gg 0$, $\exists$ a positive integer $N \ni$ for all $p, q > N$, $\rho(x_p, x_q) \ll b$.

Note that a cone metric space (X_1, ρ) is said to be complete if every Cauchy sequence is convergent.

Definition 2.7.

Let F_1 and G_1 be two self-mappings of a set X_1. If u_1` $= F_1 x_1 = G_1 x_1$ for some x_1 in X_1, then x_1 is called a coincidence point of F_1 and G_1, and u_1 is called a point of coincidence of F_1 and G_1.

Proposition 2.8.

Let F_1 and G_1 be two OWC self-mappings of a set X_1 iff there is a point x_1 in X_1 which is said to be a coincidence point of F_1 and G_1 at which F_1 and G_1 are commute.

Lemma 2.9.

Let X_1 be a set, and F_1, G_1 are two OWC self-mappings of X_1. If F_1 and G_1 have a unique point of coincidence $u_1 = F_1 x_1 = G_1 x_1$, then u_1 is said to be the unique common fixed point of F_1 and G_1.

3 Fixed Point Results

In this section we obtained affixed point result on occasionally weakly compatible for Four self- mappings in cone metric space which is a generalization, and extension of the results of [2].

Theorem. Let (Y, ρ) be a cone metric space and Q be a normal cone. And let four self mappings M, N , A and B are OWC mappings of X_1 and they satisfying the following conditions

(i) $\rho (M x, N y) \leq \alpha \rho (Ax, By) + \beta \rho(Mx, Ax) + \gamma \rho (Ny, By) +\delta[\rho (Mx, By) + \rho (Ny, Ax)]$,

for all $x, y \in Y$, for which $Mx \neq Ny$, $\alpha, \beta, \gamma, \delta \in [0, 1]$ and satisfying $\alpha + \beta + \gamma + 2\delta < 1$.

(ii) The pairs { M, A } and { N, B } are OWC.

Then, M, N, A and B have a unique common fixed point.

Proof. Given {M, A} and {N, B} are OWC, then there exists $x, y \in Y$ such that $Mx = Ax$ and $Ny = By$. We claim that $Mx = Ny$. Otherwise by (i)

$\rho (Mx, Ny) \leq \alpha \rho(Ax, By) + \beta \rho (Mx, Ax) + \gamma \rho (Ny, By) + \delta[\rho (Mx, By) + \rho(Ny, Ax)]$.

Since $Mx = Ax = w_1$ and $Ny = By = z_1$ are points of coincidence of {M, A} and {N, B}respectively.

$\Rightarrow \rho (Mx, Ny) \leq\alpha \rho (Mx, Ny) + \beta \rho (Mx, Mx) + \gamma \rho(Ny, Ny) + \delta[\rho (Mx, Ny) + \rho (Ny, Mx)]$,
$\leq (\alpha + 2\delta) \rho (Mx, Ny)$, since $\alpha + 2\delta < 1$,

$\Rightarrow \rho (Mx, Ny) < \rho (Mx, Ny)$, which is a contradiction.

Therefore, $Mx = Ny$, that is, $Mx = Ax = Ny = By$.

Moreover, if there is another point z_1 such that $Mz_1 = Az_1$, then by (i) $Mz_1 = Az_1 = Ny = By$ or $Mx = Mz_1$. Hence, $w_1 = Mx = Nx$ is a unique point of coincidence of M and A. Then by the (1.9) lemma we get that

w_1 is the unique common fixed point of M and A. …. (1).

Similarly, there is a unique point $z_1 \in Y$ such that $z_1 = Nz_1 = Bz_1$. We suppose that $w_1 \neq z_1$, then by (i) , we get that

$$\rho (w_1, z_1) = \rho (Mw_1, Nz_1) \leq \alpha \rho (Aw_1, Bz_1) + \beta \rho (Mw_1, Aw_1) + \gamma \rho (Nz_1, Bz_1) + \delta[\rho (Mw_1, Bz_1) + \rho (Mz_1, Aw_1)],$$
$$\leq\alpha \rho(w_1, z_1) + \beta \rho (w_1, w_1) + \gamma \rho(z_1`, z_1) + \delta[\rho (w_1, z_1) + \rho(z_1, w_1)] ,$$
$$\leq (\alpha + 2\delta) \rho(w_1, z_1), \text{ since } \alpha + 2\delta < 1$$

$\Rightarrow \rho(w_1, z_1) < \rho(w_1, z_1)$, it is a contradiction.

Hence, w_1 is a unique common fixed point of four self mappings M, N, A and B.

This completes the proof of the theorem.

4 Conclusion

Our result is more general than the results of [2] and it is also improvement of the results of [2]

Competing Interests

Author has declared that no competing interests exist.

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