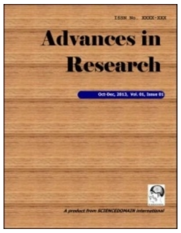

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Reliability Analysis of Steel Structures under Buckling Load in Second-order Theory

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Authors' contributions

This work was carried out in collaboration between all authors. All authors read and approved the final manuscript.

Short Research Article

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ABSTRACT

There are various theoretical methods and procedures to evaluate instability of members in steel frames, in which effects of overall buckling and also geometrical nonlinearity are considered. In the first part, aims are to compare these conventional methods and investigate the potential of the software to applying the effects in analysis. For this reason, obtained buckling loads of simple frames from conventional methods are compared with results of SAP2000. In order to consider the actual design issues and generalize the problem, buckling load of multi-storey steel frames with multiple openings (3 and 5 floors, 3 openings) is calculated. Also, uncertainties that exist in loading and modeling of structures such as geometrical imperfection, yield stress, and modulus of elasticity in buckling load of 2D framed steel structures have been studied. By performing these uncertainties to each reliability analysis procedures (first-order, second-order, and simulation methods of reliability), a reliability index from each procedure is determined. These values are studied and compared.

Keywords: Buckling; second-order theory; reliability index.

1. INTRODUCTION

Slender columns which subjected to axial loads may be susceptible to buckling type behavior. As long as the axial load on such a member is relatively small, increases in the

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load results only in axial shortening of the member. However, once a certain critical load is reached, the member suddenly bows out sideways. This bending gives rise to large deformations, which in turn cause the member to collapse. The load at which buckling occurs is thus a design criterion for compression members [1]. Also the reliability and probability of failure of steel systems with regard to the effects of uncertainties involved in parameters such as imperfect geometry, combined normal stress, yielding and the modulus of elasticity were investigated. Uncertainties in the system can be divided into two categories which include load and resistance uncertainties. Each of these uncertainties using appropriate methods have been implemented and the corresponding reliability index is obtained.

2. RESEARCH METHODS

The main purpose of this research is to evaluate computer programs capabilities in conducting second-order reliability analysis and determining buckling load (critical load) in 2D steel framed structures. For this reason and for comparison, the theoretical methods were analyzed using three different stability analysis methods and the computer programs such as SAP2000 were used for stability analysis. Initially, simple one-bay one-floor frames were analyzed using three methods. It was shown that SAP provides acceptable results for stability analysis of 2D frames. To consider actual situation, several 3 and 5 floors frames were modeled and the value of buckling forces were compared.

Then the model uncertainties including inherent uncertainties, modeling uncertainty, estimation errors, and human errors are introduced. The first and second order reliability analysis methods such as FORM and SORM are presented and various reliability indices are compared. Also, simulation methods such as Monte-Carlo and DARS have been studied and described. Finally, case studies on the basis of discussed theories are applied.

3. STABILITY ANALYSIS

The phenomenon of buckling for a complicated structure is in many ways similar to that for a simply supported strut. The behavior of a simply supported strut at the buckling load has two related characteristics. Firstly, if the strut has no initial eccentricity or applied lateral load, then the buckling load is the only load at which the strut may be in equilibrium with a sudden uncontrollable lateral deflection. Secondly, if the strut has an initial eccentricity or applied lateral load, then, according to linear elastic theory, the lateral deflection tends to infinity at the buckling load (see Fig. 1). Although the behavior of an actual strut falls into the second category, the most convenient theoretical methods of determining the buckling load rely on the assumption of no initial eccentricity or lateral load.

Even as simple a structural element as an axially loaded member behaves in a fairly complex manner. It is therefore desirable to begin the study of columns with a much idealized case, the Euler column. The axially loaded member shown in Fig. 1 is assumed to have a constant cross-sectional area and to be made of a homogeneous material. In addition four assumptions are made:

1) The ends of the member are simply supported
2) The member is perfectly straight, and the load is applied along its centroidal axis.
3) The material obeys Hooke's law

4) The deformations of the member are small enough so that the term $(y')^2$ is negligible compared to unity in the expression for the curvature $y'' / [1 + (y')^2]^3/2$. Hence the curvature can be approximated by y'' [1].

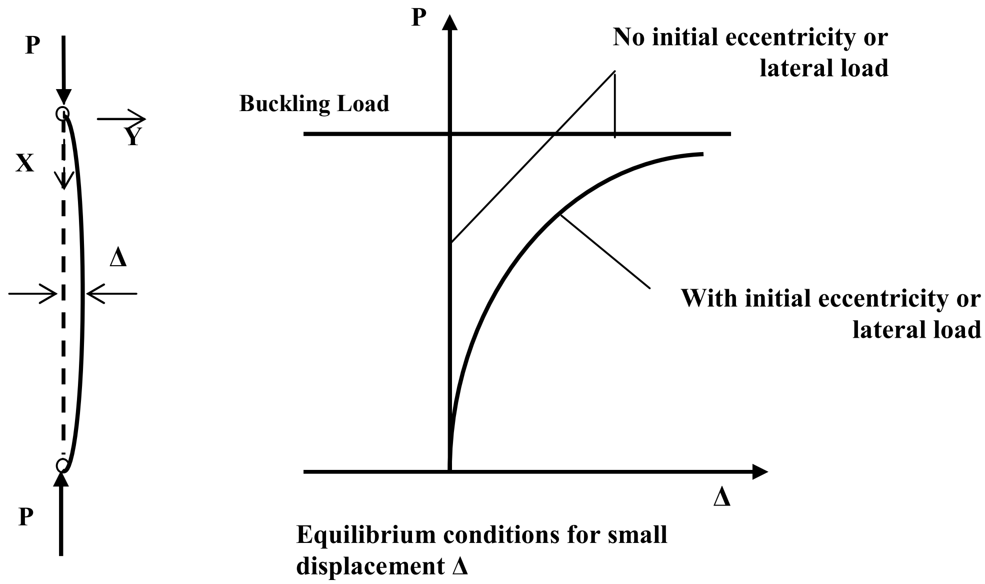


Fig. 1. Buckling behavior of simply supported strut [1]

In accordance with the criterion of neutral equilibrium the Euler load is $(\pi^2 EI) / L^2$ [2]. It is the smallest load at which a state of neutral equilibrium is possible. Hence, it is the smallest load at which the column cases to be in stable equilibrium. Up to the Euler load the column must remains straight. At the Euler load there exists a bifurcation of equilibrium that is the column can remain straight or it can assume a deformed shape of indeterminate amplitude [2].

3.1 Second-order Elastic Analysis

In geometric nonlinear or second-order elastic analysis, equilibrium is formulated on the deformed configuration of the structure. When derived on a consistent mechanics basis, this type of analysis includes both P-Δ (chord rotation) and P-δ (member curvature) effects. The P-Δ effect reduces the element flexural stiffness against side sway. The P-δ effect reduces the element flexural stiffness in both side sway and non-side sway modes of deformation. These two effects are illustrated for an arbitrary beam-column subjected to side sway in Fig. 1. Actually, few programs can model the P-δ effects precisely unless the members are subdivided into a number of elements (particularly if the members have some initial out-of-straightness). Second-order elastic analysis accounts for elastic stability effects, but it does not provide any direct information with regard to the actual inelastic strength and stability of the frame. Therefore, in any design based on this type of analysis, these aspects must be accounted for in the specification equations for member proportioning [3,4].

The majority of building structures have been designed by the elastic theory by simply choosing allowable stress values for the materials and by imposing limiting ratios such as serviceability requirements. All structures deflect under loading, but in general, the effect of this upon the overall geometry can be ignored. In the case of high-rise building, the lateral deflections may be such as to add a significant additional moment. This is known as P-Δ effect. Therefore, the governing equilibrium equations of a structure must be written with respect to the deformed geometry; the analysis is referred to as second-order analysis. On the other hand, when the lateral deflections can be ignored and the equilibrium equations are written with respect to the undeformed geometry, the analysis is referred to the first-order analysis. The load deflection behaviors of a structure analyzed by first and second order elastic methods are illustrated in Fig. 2 [5,6]. This is discussed by many authors among them Galambos [7], Allen and Bulson [8] and Chen et al. [9]. From this figure, it can be understood that the critical buckling load, needed for the evaluation of the effective length of members, may be determined by the use of either the eigenvalue analysis or the second order elastic analysis. Unlike a first-order analysis in which solutions can be obtained in a rather simple and direct manner, a second-order analysis often entails an iterative type procedure to obtain solutions. Thus, the use of eigenvalue analysis to obtain the critical buckling load is the simplest way [10].

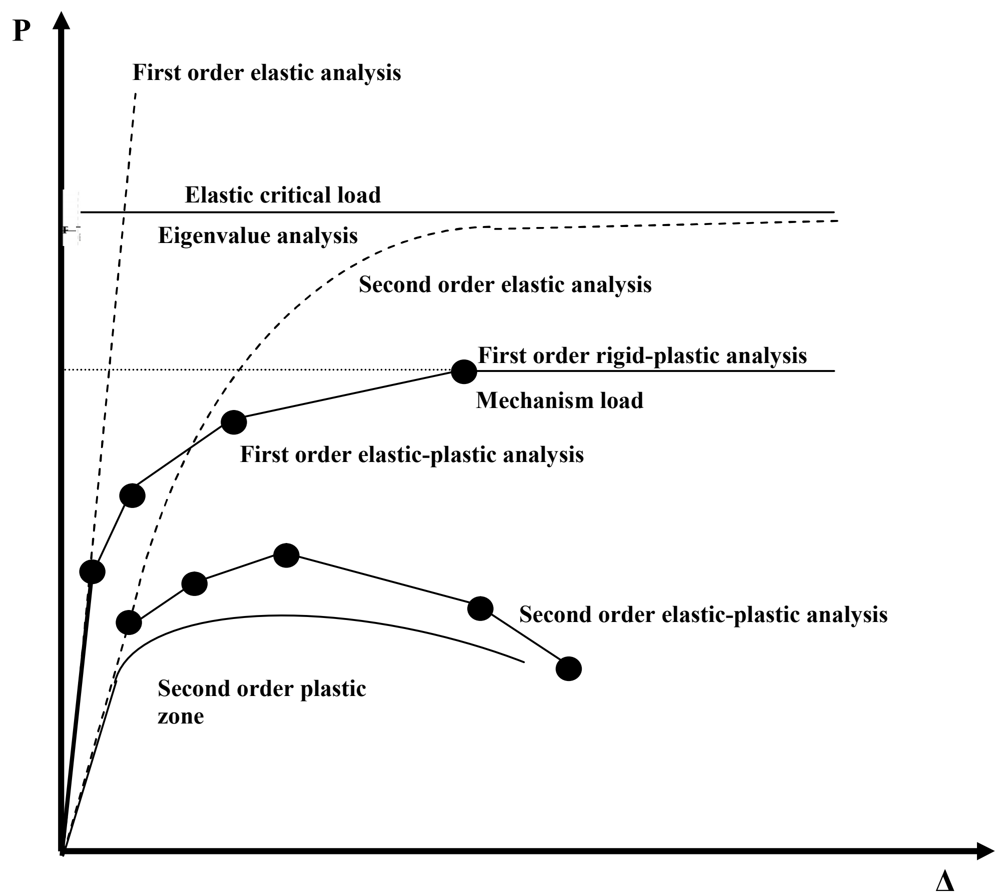
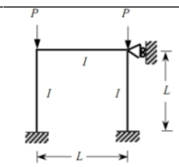
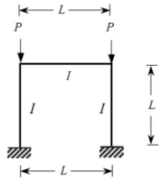
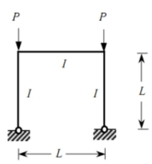
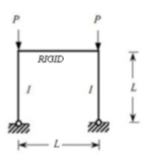


Fig. 2. Load displacement curve [2]

3.2 Stability Analysis of Simple Frames

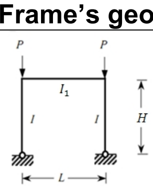
Table 1. Comparison of results of stability analysis of simple frames

Frame's geometry		Methods			
		Theory		Mahfouz's method [3]	SAP2000
1)		Chajes [1] and Renton [11]	$P_{cr} = 25.2\frac{EI}{L^2}$ $\rho = \frac{P_{cr}}{\pi^2 EI/L^2} = 2.55329$	$\rho = 2.554$	$\rho = 2.5581$
2)		Chajes [1] and Renton [11]	$P_{cr} = 7.34\frac{EI}{L^2}$ $\rho = \frac{P_{cr}}{\pi^2 EI/L^2} = 0.7437$	$\rho = 0.7475$	$\rho = 0.7478$
3)		Timoshenko and Gere [12]	$P_{cr} = 1.82\frac{EI}{L^2}$ $\rho = \frac{P_{cr}}{\pi^2 EI/L^2} = 0.1844$	$\rho = 0.1843$	$\rho = 0.18454$
4)		Timoshenko and Gere [12]	$P_{cr} = \frac{\pi^2 EI}{4L^2}$ $\rho = \frac{P_{cr}}{\pi^2 EI/L^2} = 0.25$	$\rho = 0.2499$	$\rho = 0.25$

SAP2000 verification (Table 1)

Frame No. 3- A one-story, one-bay, two-dimensional frame is subjected to an axial force, P, at the top of each column. The buckling load for this configuration is calculated and compared with independent results calculated using formulas derived in Timoshenko and Gere 1961 (see Table 2) [13].

Table 2. Sample SAP2000 verification for case No.3

Frame's geometry	Section properties	Material properties
	$A = 7.81\ e-03\ m^4$ $I = I_1 = 5.696\ e-03\ m^4$ $and\ H = L = 3.6576m$	$E = 2.0389019\ e + 07$

Independent results are calculated using formulas presented in Article 2.4 on pages 62 through 66 in Timoshenko and Gere (1961):

$$P_{cr} = \frac{(kl)^2 EI}{h^2}$$

$$kl \tan kl = \frac{6I_1 h}{IL}$$

According to Table 2, $I = I_1$ and $H = L$

$$kl \tan kl = 6$$

By trivial and error using excel:

$$kl = 1.34955282$$

So,

$$P_{cr} = \frac{(1.34955282)^2\ (2.0389019e + 07)(5.696e - 03)}{3.6576^2}$$

$$P_{cr} = 158.108\ ton$$

4. EXAMPLES

According to the results of stability analysis of SAP2000 (based on Timoshenko) and FORTRAN (based on the direct method) [3] (see Table 1) that substantially comply with each other, so we can use SAP2000 as a tool for stability analysis and the second-order analysis.

In the following examples (see Fig. 3 through 6), two actual steel frames with various dimensions and sections will be analyze in terms of stability and second-order and buckling force can be determined.

4.1 Example One- Three Stories, Three Openings Frame (Figs. 3 and 4)

In order to consider actual problems and also to compare buckling factors (or critical loads) of various structures, several steel framed structures had been analyzed under stability analysis mode in SAP2000 (Tables 5 and 6).

At first step, the specific steel framed structure has been designed based on AISC-LRFD 93 specification with linear elastic analysis manner. Then they were analyzed in SAP2000 (under the same condition as ETABS) under elastic buckling analysis to obtain buckling factors (Tables 3 and 4).

Table 3. Results comparison

Model	Output parameter	SAP2000	Independent
A 1 element per object	Buckling Load (ton)	158.58586	158.108

Table 4. Material properties and boundary conditions for example one and two

Material name	Modulus of elasticity (ton/m^2)	Poisson's Ratio	Mass per unit vol.	Fy (ton/m^2)	Fu (ton/m^2)	Fc (ton/m^2)
Steel	20400000	0.3	8.0000E-01	24000	40000	-
Concrete	2531050.650	0.2	2.4480E-01	-	42184.178	2812.279
Other	20389019.158	0.3	7.9814E-01	-	-	-

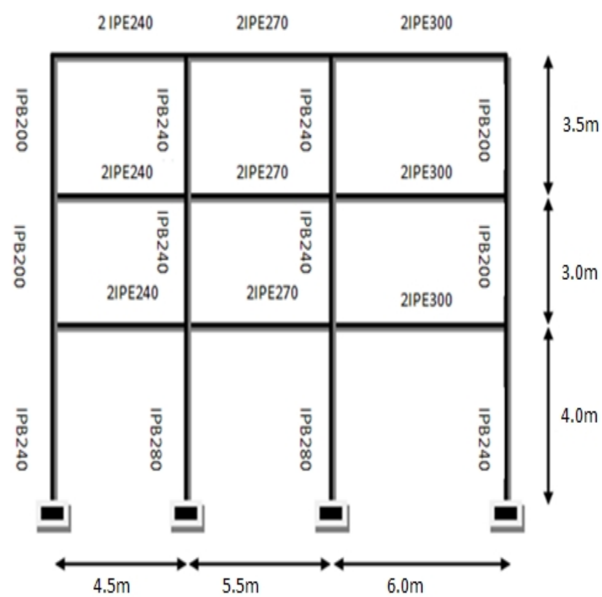


Fig. 3. Dimensions and assigned frame sections of example 1 (3 stories frame)

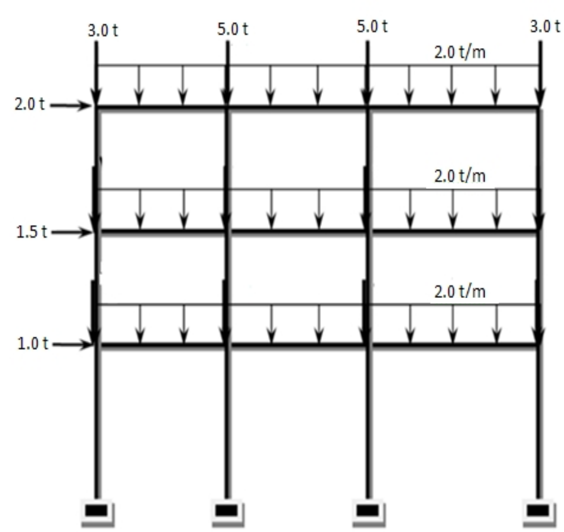


Fig. 4. Frame loading of example 1 (3 stories frame)

Table 5. Buckling force of the frame in different modes (3 stories frame)

Output case	Step type	Step no.	Buckling load
Text	**Text**	**Unit less**	**Ton**
BUCK1	Mode	1	16.23809018
BUCK1	Mode	2	26.42317921
BUCK1	Mode	3	40.72029836
BUCK1	Mode	4	55.64709977
BUCK1	Mode	5	63.82292344
BUCK1	Mode	6	69.63329296

Table 6. Buckling force of the frame in different modes

Output case	Step type	Step no.	Buckling load
Text	**Text**	**Unit less**	**Ton**
BUCK1	Mode	1	10.28342643
BUCK1	Mode	2	13.09279733
BUCK1	Mode	3	19.86167554
BUCK1	Mode	4	25.09709932
BUCK1	Mode	5	34.96110242
BUCK1	Mode	6	36.52135178

4.2 Example Two- Five Stories, Three Openings Frame (Figs. 5 and 6)

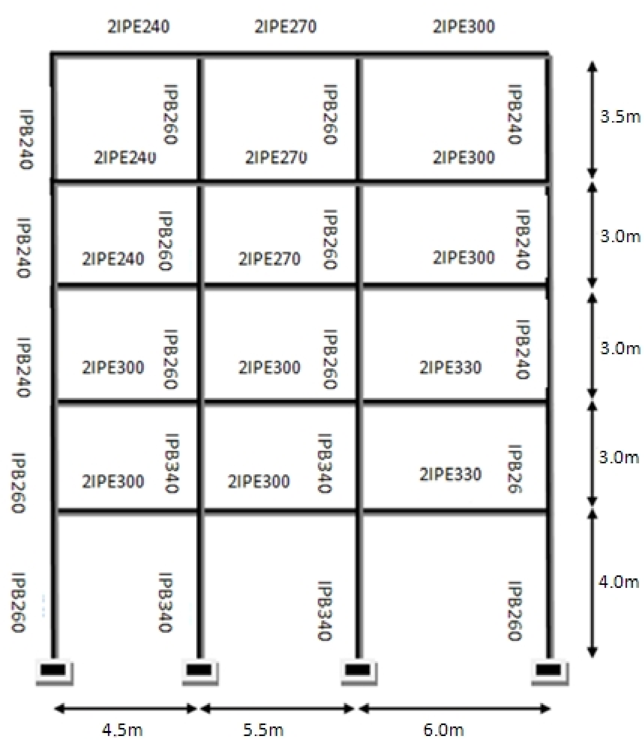


Fig. 5. Dimensions and assigned frame sections of example 2 (5 stories frame)

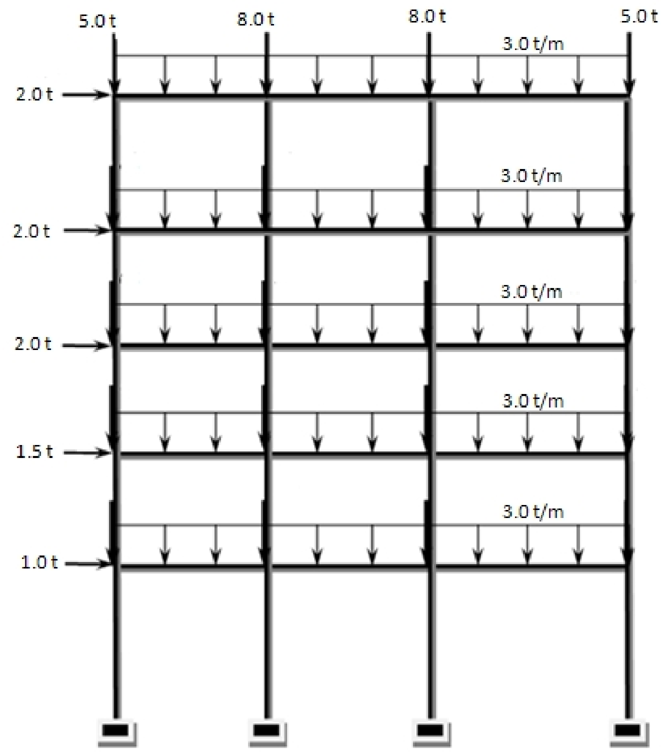


Fig. 6. Frame loading of example 2 (5 stories frame)

According to the results of buckling analysis of frames and comparison (based on SAP2000 analysis, independent method and FORTRAN) it can be seen that in the two-dimensional steel frames, SAP2000 provides high accuracy in second-order analysis. It is also possible to perform this analysis to the actual steel frames. Due to the lack of accuracy in loading and modeling and also in fabricating buildings, there is a high demand for software or specific method by which you can perform these inaccuracies and uncertainties. Also, effects of uncertainties in structures for evaluation of the resistance and load bias factors have remarkable influence in obtaining the overall reliability of the structural frame system under buckling load and should be consider in the actual condition and under strict observation. The reliability analysis of structures with regard the buckling shape and emphasis on second-order analysis provides a relatively good point of action on the subject.

5. RELIABILITY ANALYSIS

5.1 Introduction

In this section, the aims are to evaluate the reliability and failure probability of steel systems considering the effects of uncertainties such as imperfect geometry, the yield stress and elastic modulus. Generally, in the steel systems, these uncertainties are classified into two categories: uncertainty of load and resistance. Each of these, with regard to appropriate methods has been analyzed and the corresponding reliability index is obtained. At the end, the obtained indexes are compared and the effects of these uncertainties are illustrated [14].

The issues of structural reliability analysis in which the latter uncertainties are analyzed, were studied, at first in terms of an n-dimensional vector of random variables. For providing appropriate information about geometrical properties, elastic stiffness and strength, these variables taken random into account. Therefore any structure or system properties can be loaded as a point in a limited space to interpret by the variable$\{x_i\}_{i=1}^{n}$.

The main attention should be focus to a local failure criterion which is determined explicitly through a performance function g(x). We present two specific areas g(x) <0 and g(x)> 0 in the space$\{x_i\}_{i=1}^{n}$, respectively, indicating the safe and unsafe areas. Surface g(x) =0 is the boundary of the secure and non-secure areas. In this situation the failure probability P_f known as given below:

$$\boldsymbol{P}_{\mathbf{f}} = \int_{\mathbf{g(x)}} \mathbf{P_x(x)dx} \tag{1}$$

Where P_f is the probability density function of the n-dimensional vector of random variables $\{x_i\}_{i=1}^{n}$ [15].

5.2 Models of Uncertainties

Sources of uncertainty in engineering problems can be divided into four categories:

- Inherent or physical uncertainties
- Model uncertainties
- Estimation errors
- Human errors

Detailed evaluation of failure probability P_f ($P_f = \int_{g(x)} P_x(x)dx$) when g (x) is a linear function of x in the following form is possible:

$$\mathbf{g(x)} = \mathbf{a_0} + \sum_{\mathbf{i=1}}^{\mathbf{N}} \mathbf{a_i X_i} \tag{2}$$

And all random variables of $\{x_i\}_{i=1}^{N}$ are normal. The mean vector is μ and covariance matrix is called [c]. In this case we can show that: [14,15]

$$\boldsymbol{P}_{\mathbf{f}} = \boldsymbol{\phi}(-\boldsymbol{\beta}) \tag{3}$$

Where

$$\boldsymbol{\beta} = \frac{\mathbf{a_0} + \sum_{\mathbf{i=1}}^{\mathbf{n}} \mathbf{a_i \mu_i}}{\sqrt{\sum_{\mathbf{i=1}}^{\mathbf{n}} \sum_{\mathbf{j=1}}^{\mathbf{n}} \mathbf{a_i a_j C_{ij}}}} \tag{4}$$

5.2.1 First-order reliability method (FORM)

Cornell (1969) proposed the non-linearity of g(x) in x parameters and independency of normal random variables. With the assumption that g(x) can be derived by its average vicinity, g(x) has been replaced by its first-order Taylor expansion around μ. Based on this study, the reliability index can be determined as follows: [15].

$$\beta_{FOSM} = \frac{g(\mu_1,\mu_2,\dots,\mu_n)+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial^2 g}{\partial x_i \partial x_j}(\mu_i,\mu_j)<(X_i-\mu_i)(X_j-\mu_j)>}{\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial g(\mu_i)}{\partial x_i}\frac{\partial g(\mu_j)}{\partial x_j}<(X_i-\mu_i)(X_j-\mu_j)>} \quad (5)$$

5.2.2 Second-order reliability method (SORM)

The following four categories of issues to be considered in evaluating of P_f:

1- g(x) is linear in terms of x and x is a vector of Gaussian random variables.
2- g(x) is linear in terms of x and x is a vector of the variables is non-Gaussian.
3- g(x) is nonlinear in terms of x and x is a vector of Gaussian random variables.
4- g(x) is nonlinear in terms of x and x is a vector of the variables is non-Gaussian.

The first category of the above problems has exact dependent solutions and remaining issues are generally needed to estimate. Moreover, problems in category (2) and (4) can always be transformed into category 3 issues. In FORM, g(x) by first-order Taylor expansion around a given point is replaced thereby the performance function is linear. This is a reasonable estimate of the probability damage in which g(x) is a linear analysis around the point. The question is : How can we improve the first-order approximation, leads to the development of the second-order reliability methods. The main idea of SORM is to replace the performance function G(X) by a second-order Taylor expansion around the point, which follows:

$$\mathbf{g}(\mathbf{x}) = \mathbf{g}(\mathbf{x}^*) + (\mathbf{x}-\mathbf{x}^*)^{\mathbf{T}}\mathbf{g}_{\mathbf{x}}(\mathbf{x}^*) + \frac{1}{2}(\mathbf{x}-\mathbf{x}^*)^{\mathbf{T}}\mathbf{G}_{\mathbf{x}}(\mathbf{x}^*)(\mathbf{x}-\mathbf{x}^*) \quad (6)$$

Where g_x is gradient vector of g(x) and G(X) in the n × n matrix of second derivatives of g(x). This results in a parabolic surface of failure and ongoing operation of multi-normal probability integral includes an evaluation of the failure surface. There are two strategies for evaluation of parabolic surface. The first involves the implementation of original curves of limit state with proposed parabolic surface at the point. The second method is to use a point evaluation strategy. Fig. 7 presents geometric interpretation of the SORM [10].

5.3 Simulation Methods

In the process of making predictions about some physical systems it is required to abide by the following four steps:

A) Observation of physical system;
B) Formulation of hypotheses;
C) Predict system behavior based on the assumption;
D) Conduct experiments to demonstrate the acceptability of the hypothesis.

The method described here is one of the fundamental simulation methods in the reliability the Monte-Carlo method [15].

5.3.1 Monte-carlo method

When high-speed numerical computing facilities are available, the Monte-Carlo method is a simple procedure that can often useful to gain distribution of$F_R(r)$. If R is a function of n independent random variables of Y_i , we have:

$$\mathbf{R} = \mathbf{g}(\mathbf{Y_1}, \mathbf{Y_2}, \dots, \mathbf{Y_n}) \tag{6}$$

This method consists of three steps. [15]

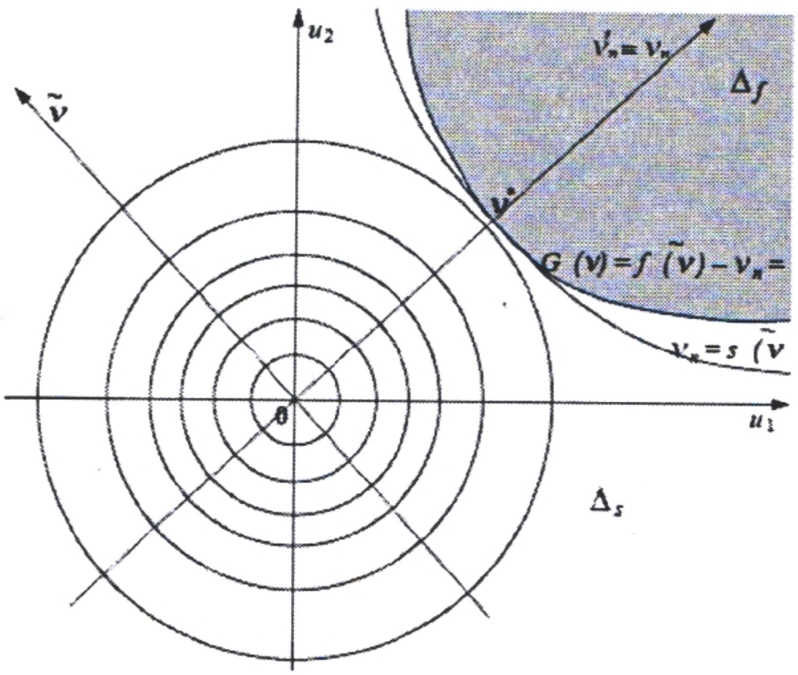


Fig. 7. Geometric interpretation of the second-order reliability analysis SORM [10]

5.3.2 DARS method

Consider a limit state being characterized by a limit state function Z = g(X), so that failure may be associated with g(X) < 0. The vector X = {X1, X2 ...Xn} represents the set of random variables for which probability distributions are known. We are looking for the probability of failure:

$$P\ \{Failure\} = P\ \{g(X) < 0\} \tag{7}$$

The DARS (DIRECTIONAL ADAPTIVE RESPONSE SURFACE SAMPLING) method to solve this problem is carried out in the so called U-space. That is, every random variable X is transformed into a standard normal variable U. For independent variables this can be done by:

$$U = \varphi^{-1}\{F(X)\} \tag{8}$$

Where F (...) is the distribution function for X and $\varphi(..)$ is the distribution for the standard normal variable U. In the case of dependent variables the Rosenblat transformation is to be used.

When finite element analysis is combined with methods of structural reliability, computation time increases. Particularly, in the non-linear analyzes such as Monte-Carlo, Direct Sampling and Numerical Integration, variable reduction techniques such as Importance Sampling requires prior knowledge about the critical areas that are not accessible. Where computations are not time-consuming, FORM is applicable [16].

5.4 Evaluation of the Reliability Index

First, some functions required for structural buckling analysis are presented. Then, reliability analysis is performed according to these functions. Some functions according to the first-order, second-order, or Monte Carlo methods will be different. Probability of failure and reliability index obtained from the analysis of the functions will be compared.

5.4.1 Definition of limit state functions

LSF1: g (X) = R (X) - S (X)
LSF2: g (X) = XR.R (X) - XS.S (X)
R (X) = frame resistance function
S (X) = frame actual load function
XR = coefficient BIAS of frame resistance
XS = coefficient BIAS of frame load [14].

5.5 Uncertainty Modeling

The uncertainty in evaluation of the above limit state functions LSF1 and LSF2 may be due to the structural buckling resistance R(X) and the load modeling S(X) as well. The coefficient of variation of R(X) and S(X) are denoted by COV(R) and COV(S). COV(R) may also be dependent on fabrication of structural material as well as building procedures etc.

The uncertainty in evaluation of the resistance and load bias factors also may play a significant role in determining the overall reliability of the structural frame system under buckling load. The uncertainty in the structural system resistance may be due to both geometrical and also material parameters of structure as well as calculation model and also numerical errors involved in the analysis. The uncertainty in the geometrical parameters may include fabrication imperfection and load eccentricity. The uncertainty in the material parameters might also include yield stress and modulus of elasticity. The practical range of COV(R) is usually taken in the range of 0.05 to 0.15 [10]. But for the sake of sensitivity analysis this is varied up to 0.3. As shown in the Figs. 8 and 9, the most important results and the most reliable part of diagram is between 0.05 to 0.15 where reliability index varied from 3.86 to 3.25 and after that trends decrease steadily until 0.3.

6. RESULTS AND DISCUSSION

Two introduced, three-story and five-story frames in previous examples are analyzed by three different methods, first-order reliability, and second-order reliability and simulation methods respectively.

6.1 Evaluation of the Reliability Index of Frame's Columns

A - Internal columns

Table 7. Reliability index (P_f) and probability of failure (B) results based on different methods

Reliability method	P_f	B	No. of g(X)
FORM (LSF1)	0.00006	3.84447	31
SORM (LSF1)	6.0407e -5	3.8445	-
FORM(LSF2)	0.0014162	2.9854	46
SORM (LSF2)	0.0013553	2.9988	-
Monte Carlo(LSF1)	1.517e -4	3.6124	1e+7
Monet Carlo(LSF2)	0.0018	2.9138	1e+7

Table 8. Importance factors of load (IM. R(x)) and resistance (IM.S(X)) based on FOSM

Reliability method	IM. R(x)	IM.S(X)
FOSM(LSF1)	6.402e -1	-7.682e -1
FOSM(LSF2)	5.037e -1	-5.742e -1

Table 9. The mean and standard deviation of the function g (x)-frame results: FOSM

Reliability method	Mean g(x)	Std. g(X)
FOSM(LSF1)	0.302	0.096354
FOSM(LSF2)	0.290	0.07416

Table 10. Reliability Index values of column based on changes in slender ratio in the second-order theory based on FORM & SORM (see Figs. 8 and 9)

Reliability method	λ =100	λ = 125	λ = 150
FORM (LSF1)	4.76402	2.91648	0.56324
SORM (LSF1)	4.7640	2.9165	0.56324
FORM(LSF2)	3.6579	2.3137	0.61845
SORM (LSF2)	3.6803	2.3425	0.58904

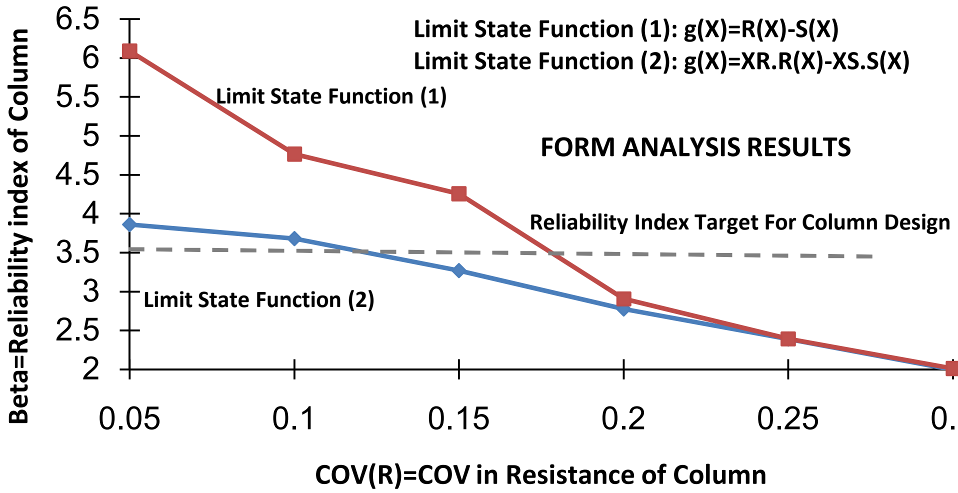


Fig. 8. Sensitivity analysis of reliability index of column with λ = 100 FORM

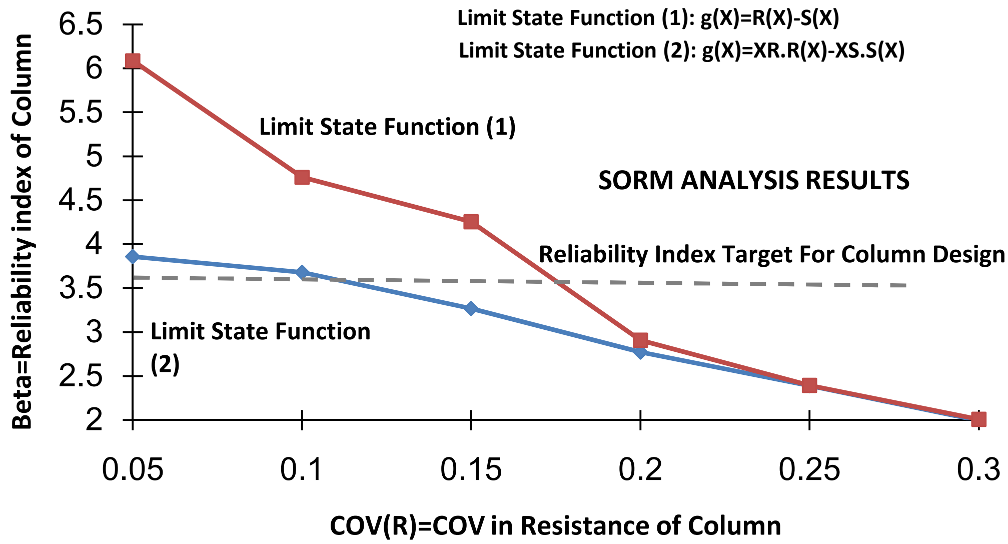


Fig. 9. Sensitivity analysis of reliability index of column with λ = 100 SORM

6.2 Evaluation of the Reliability Index of Columns Frame:

A - External columns

Table 11. Reliability index (P_f) and probability of failure (B) results based on different methods

Reliability method	Pf	B	No. of g(X)
FORM (LSF1)	5.216 e -007	4.8833	88
SORM (LSF1)	7.8175 e -009	5.6544	-
FORM(LSF2)	0.00006	6.27041	31
SORM (LSF2)	1.8005 e -010	6.2704	-

Table 12. Importance factors of load (IM. R(x)) and resistance (IM.S(X)) based on FOSM

Reliability method	IM. R(x)	IM.S(X)
FOSM(LSF1)	0.6257	-0.4128
FOSM(LSF2)	0.8137	-0.5812

Table 13. The mean and standard deviation of the function g (x)-frame results: FOSM

Reliability method	Mean g(x)	Std. g(X)
FOSM(LSF1)	0.53	0.11214
FOSM(LSF2)	0.52	0.0818535

Results of reliability analysis of steel structures (case study) in terms of buckling of columns (in the second order theory) suggest these following points:

- Reliability index of frame based on the first limit state function (LSF1) based on FORM and SORM methods was above the reliability target for beams - columns with regard to LRFD method (Tables 7 and 11).
- Reliability index of frame based on the Monte-Carlo method is slightly lower than reliability target for beams - columns in LRFD method (Tables 7 and 11).
- Reliability index of structures have been studied in terms of buckling under the second part of LSF1 is considerably lower than the reliability target for beams - columns in LRFD method (Tables 7 and 11).
- Coefficients of importance of structural limit resistance function R (X) are lower than the coefficients of importance of structural limit load function S (X) based on both limit state functions of reliability (Tables 8 and 12).
- The mean and standard deviation of the initial β in the second part of limit state function is lower than the first part (Tables 9 and 13).
- Reliability index of the column was studied with λ = 100 which λ varies from 100 to 150 indicate that reliability index for λ = 100 are higher than the reliability target for beams-columns in LRFD method (Table 10).
- But such column with λ = 100 in increased slenderness ratio to 150 is near the limit of annual failures (Table 10).
- The column with λ = 125 represents a relatively acceptable annual safety margin in terms of stability (Table 10).

7. CONCLUSION

Results of reliability analysis of structural buckling of columns under the FORM and SORM methods in terms of variation coefficients of slenderness ratios λ were investigated in the range of 100-150, which gives the reliability index β in the range of 3.0 to 3.5 for buckling about the x-axis and y-axis respectively. Results of structural reliability analysis of the first-order and second-order methods about studied columns with mean λ = 100 in the range λ = 75-125 represents variations of reliability index in the range of 2.32 to 4.33 for the second limit state function ranging from 2.92 to 6.27 for the first limit state function.

Sensitivity analysis is performed based on changes in COV(R) ranging from0.05 to 0.3 represents a sharp downward change in reliability index of columns subjected to inelastic buckling range of COV(R) = 0.05-0.15, and the upper limit of the target for the β= 3.8 varied in the range of 3.2 to 3.6 with COV(R) = 0.15. Given the range of variation COV(R), which is probably 0.1 to 0.15, β is in the acceptable range in terms of design. Bias in terms of resistance and load parameters, as was observed in the range of β has been greatly reduced but is still close to β_{target}. Sensitivity studies indicate that the limit state function study shows more realistic response. Bias coefficient of β is in the range of COV(R) = 0.05-0.1.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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