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The Price of Asset-liability Control under Tail Conditional Expectation with No Transaction Cost

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Research Article

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Abstract

Asset-liability management is a means of managing the risk that can arise from the changes in the relationship between assets and liabilities. Value-at-risk (V_aR) and tail conditional expectation (TCE) have also emerged in recent years as standard tools for measuring and controlling the risk of trading portfolios. In some dynamical settings however, the limits of TCE can be transformed into the limits of V_aR and conversely even though TCE is more preferable to V_aR since it is coherent and V_aR is not. In this paper we obtain the optimal price of an institution's assets- liabilities under the TCE with no transaction cost.

2010 Mathematics Subject Classification: 91G10, D92, C61, 37N40.

Keywords: Risk management; Tail conditional expectation; Asset-liability control; optimal price; Value-at-risk; Financial institution;

1 Introduction

In cases such as in portfolio containing option as well as credit portfolio (i.e wealth distributions that are highly skewed), it is reasonable to consider asymmetric risk measures since individuals are typically loss averse. Asset-liability control is a means of managing the risk that can arise from changes in the relationship between asset and liabilities. Value – at – Risk (V_aR), a downside risk measure, has also emerged as the industry standard with regulatory authorities enforcing its use in risk measurement and management (Daniel et al., 2009; Jorion, 2001).

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Let risk Z be a non-negative random variable with cumulative distribution F, where Z may refer to a claim for an institution's asset or liability. Given $0 < \alpha < 1$, the z_α, determined by $\bar{F}(z_\alpha) = 1 - F(z_\alpha) = \alpha$ and denoted by $V_aR_z(1-\alpha)$ is called the value at risk (V_aR) with a degree of confidence 1 – q. The conditional expectation of Z given by $Z > z_\alpha$, denoted by $TCE_z(z_\alpha) = E\big((Z|Z > z_\alpha)\big)$ is called the a tail conditional expectation (TCE) of Z at V_aR z_a .

Notice that

$$TCE_Z(z_\alpha) = z_\alpha + E(Z - Z_\alpha | Z > z_\alpha),$$

where $(Z - t|z > t)$is known as the residual lifetime in reliability (Shaked and Shanthikumar, 1994) and the excess loss (liability) in finance.

The $TCE_Z(z)$ function is increasing in $z > 0$ or equivalently, $TCE_Z(z)$ is decreasing in $\alpha \in (0,1)$ since $\frac{d}{dz}\big(z + E(Z - z|Z > z)\big) \geq 0$.

Both V_aR and TCE are important measures of right – tail risks which frequently encountered in the insurance and financial investment. It is known that the TCE satisfies all the desirable properties of a coherent risk measure (Artzner et al., 1999; Daniel et al., 2010; Rockafellar and Uryasev, 2001), and that the TCE provides a more conservative measure of risk than V_aR for the same level of degree of confidence (Landsman and Valdez, 2003). Therefore, the TCE is more preferable than the V_aR in many applications and has recently received growing attentions in the insurance and finance literature. However in some dynamical settings, it is possible to transform a TCE limit into an equivalent V_aR limit, and conversely (Cuoco et al., 2008).

In this paper we apply TCE to the asset-liability control model to determine the price of asset or liability of a financial institution without transaction cost.

2 Formulation of the Problem

We assume the institution operates on a market of one riskless bank with constant interest rate r and m different stock. The evolution of stock prices is described by an m-dimensional Wiener process $W(t)$ on the filtered probability space $(\Omega, f, (f_t), P)$ with $f_t = \sigma\{W(s); 0 < s < t\}$:

$$dB(t) = rB(t)dt \qquad (1)$$

$$dS(t) = \mu_i S(t)dt + \sigma_i S(t)dW(t), \;\; i = 1, \dots, m\,. \qquad (2)$$

Here $\sigma_t = \left(\sigma_{ij}(t)\right)_{i\leq j\leq m}$ is an m x m positive definite matrix representing the covariance structure, $\sigma'\sigma$. Where σ' is the transpose of σ. The institution has initially x_0 Naria invested in the bank and $(x_{1,\dots,}x_m)$ Naira invested in stock $1,\dots,m$. It can control its portfolio composition by buying and selling arbitrarily large or small amounts of stock from its bank account at any time. The institutions portfolio selection strategy θ is described by the control processes $C\,(t)$ and $L\,(t)$, where $C\,(t)$ (the institution net cash flow at time t) and $L(t)$ (the market value of the institution's liabilities at time t) are $f-$ adapted vector processes. The dynamics of the control system (Osu and Ihedioha, 2011) is governed by the differential equations:

$$dS(t) = S(t)\left[\left(\mu + \frac{\sigma^2}{2}\right)dt + \sigma dW(t)\right] + dC(t) - dL(t) \tag{3}$$

and

$$dB(t) = rB(t)dt - (1+\alpha)dC(t) + \left(1 - \lambda\right)dL(t), \tag{4}$$

with boundary conditions $S(t) = S_t, S(0) = S_0$ and $B(t) = B_t,\ B(0) = B_0$.

Defined a wealth process $h(t)$ as a sum additive random and multiplicative terms thus:

$$h(t) = \begin{cases} \left(1 - \lambda\right)S(t) + B(t), & with\,probability\;\; q \\ (1-\lambda)S(t)B(t), & with\,probability\; 1-q \end{cases}, \tag{5}$$

where λ is a stochastic positive factor with probability distribution $\pi(\lambda)$, such that with probability q the integral form of (3) and (4) combined is

$$h(t) = h(0) + \int_0^t \left[rB + (1-\lambda)\left(\mu + \frac{\sigma^2}{2}\right)\right]ds \\ +(1-\lambda)\sigma\int_0^t dW(s) + (\lambda+\alpha)C(t) \tag{6}$$

Assume $\lambda \to 0$ and $\alpha \to 0$, (that is the Merton (1969, 1971) analysis of no transaction and no consumption), we get

$$h(t) = h(0) + \int_0^t \left[rB + \left(\mu + \frac{\sigma^2}{2}\right)\right]ds \;\; +\sigma\int_0^t dW(s). \tag{7}$$

The processes $C(t)$, $L(t)$ and hence $h(t)$ are right continuous with left limit at each $t \geq 0$. For each available strategy (C, L), we can associate a feasible set of controls of the long term performance functional

$$\mathfrak{J}_z(C, L) = \lim_{t\to\infty}\frac{1}{t}E_x[\ln(h(t))] \tag{8}$$

with $z = (C, L), z \in \Re_+^2$. The objective is to optimize the long-run rate of growth

$$V(x, y) = \sup_{(C,L)\in\mathfrak{B}}\ \mathfrak{J}_z(C, L). \tag{9}$$

$\mathfrak{B}$ is a class of pair $(x, y)\epsilon\, z$, where x and y are the initial endowment of the riskless and risky asset respectively.

Let (C, L) be any feasible policy. These set of controls can be approximated by a sequence of continuous processes (C_n, L_n) such that for h_n the net wealth corresponding to them, we have;

$$\lim_{t\to\infty} \inf \frac{1}{t}[\ln h(t)] \leq \lim_{n} \lim_{t\to\infty} inf \frac{1}{t}[\ln h_n(t)]. \tag{10}$$

Thus, we can softly assume (C_n, L_n) such that wealth corresponding to them, we have $C_0 = L_o = 0$.

Let $\ln[h(t)]$ relate to the processes $B(t)$ and $S(t)$ (using Ito's formula) by

$$\ln[h_t] = \int_0^t \frac{1}{h_s}\left[rB(s) + \left(1-\lambda\right)\left(\mu + \frac{\sigma^2}{z}\right)S(s)\right]ds + \left(1-\lambda\right)\sigma\int_0^t \frac{S(s)}{h_s}dw(s)_{(5)} - \left(\lambda - \alpha\right)\int_0^t \frac{dz(s)}{h_s}$$

(Rodriguez, 2005), by the assumption above, we have

$$\ln[h(t)] = \int_0^t \frac{1}{h_s}\left[rB(s) + \left(\mu + \frac{\sigma^2}{2}\right)S(s)\right]ds + \int_0^t \frac{S(s)}{h_s}\sigma dw(s) \tag{11}$$

or

$$\mathrm{h_t} = \mathrm{h_0}\exp\int_o^t\left(\left(rB(s) + \mu S(s) + \frac{1}{2}\sigma^2 S(s)\right)ds + \int_0^t S(s)\sigma dw(s)\right),$$

where $h_0 > 0$ denote the initial value of the portfolio. Note that (11) implies

$$\mathrm{h(t+\tau)} = \mathrm{h(t)}\exp\left(\int_t^{t+\tau}\left(rB(s) + \mu S(s) + \frac{1}{2}\sigma^2 S(s)\right)ds + \int_t^{t+\tau} S(s)\sigma dw(s)\right), \tag{12}$$

for any $\tau > 0$.

For a given $\tau > 0, h > 0$ *and* $S \in R^n$, let

$$\hbar_{t+\tau}(\mathrm{h_t}, \mathrm{S}) = \mathrm{h_t}\exp\left(\left(rB(s) + \mu S(s) + \frac{1}{2}\sigma^2 S(s)\right)\tau + S\sigma\big(w(t+\tau) - w(t)\big)\right). \tag{13}$$

For a given probability level $\alpha \in (0,1)$ and a given horizon $\tau > 0$, the V_aR at time t of a portfolio $s \in S$, denoted by $VaR_t^{\alpha,S}$ is then given by

$$VaR_t^{\alpha,S} = inf\{L \geq 0 : P\big(h_t^S - \hbar_{t+\tau}(h_t^s, S_t)\big) \geq L \big| f_t < \alpha\} = \left(Q_t^{\alpha,S}\right)^- , \tag{14}$$

where

$$Q_t^{\alpha,S} = sup\{L \in R^n : P(\hbar_{t+\tau}(h_t^s, S_t) - h_t^S) \geq L \big| f_t < \alpha\}$$

is the quantile of the projected asset gain over the interval $(t, t+\tau)$ and $z^- = \max[0, -z]$. In other words, $VaR_t^{\alpha,S}$ is the liability over the next period of length τ which would be exceeded only with a (small) conditional probability α if the current price S_t were kept unchanged?

The fact $VaR_t^{\alpha,S}$ is computed under the assumption that the current portfolio is kept unchanged reflects the actual practice and the fact that the financial institutions monitoring their traders do not typically know the trades' future portfolio choices over V_aR horizon. The measure of V_aR in (14) only requires the knowledge of the current portfolio value, the current asset value and the conditional distribution of asset returns.

The TCE of a price $s \in S$ is defined by

$$TCE_t^{\alpha,S} = \left(\frac{E\left[h_t^S - \hbar_{t+\tau}(h_t^S, S_t)\right]_{h_t^S - \hbar_{t+\tau}(h_t^S, S_t) \geq -Q_t^{\alpha,S}}}{\alpha} \middle| f_t \right)^+, \tag{15}$$

where $z^+ = \max[0, z]$.

Proposition 1

We have

$$VaR_t^{\alpha,S} = h_t^S \left[1 - exp\left(\left(rB(t) + S_t\mu + \frac{1}{2} S_t\sigma^2\right)\tau + N^{-1}(\alpha) S_t \sigma \sqrt{\tau}\right)\right]^+ \tag{16}$$

and

$$TCE_t^{\alpha,S} = h_t^S \left[1 - exp \frac{\left((rB + (S_t, \mu)\tau + NN^{-1}(\alpha) + S_t\sigma\sqrt{\tau}\right)}{\alpha}\right]^t \tag{17}$$

Where $N(x)\ and\ N^{-1}(x)$ denote the normal distribution and inverse distribution functions.

Proof:

We have

$$\begin{aligned} & P\left(\hbar_{t+\tau}(\mathrm{h_t},\ S_t - \mathrm{h_t}) \leq \left|f_t\right)\right) \\ & \quad = P\left(exp\left(rB(t) + S(t)\mu + \frac{1}{2} S_t \sigma^2 + S_t\sigma(w_{t-\tau} - w_t) \leq 1 + \frac{L}{\mathrm{h_t}} \middle| f_t\right)\right) \\ & \quad = P\left(S_t\sigma(w_{(t-\tau)} - w_t) \leq log\ \left(1 + \frac{L}{h_t}\right)^+ - \ (rB(t) + S_t\mu + \frac{1}{2}\ S(t)\ \sigma^2)\tau \middle| f_t\right) \\ & \quad = N\left(lo\,g\left(1 + \frac{L}{\mathrm{h_t}}\right)^+ - (rB + S_t\mu + \frac{1}{2}\ \mathrm{S_t}\ \sigma^2)\tau\right) \end{aligned}$$

The last equation is due to the fact that the random variable $S(t)\sigma(w_{(t+\tau)} - w_t)$ is conditionally normally distributed with mean 0 and variance $S(t, \sigma^2)\tau$.

Thus

$$\begin{aligned} & P\left(\hbar_{t+\tau}(h_t), \qquad S_t - h_t \leq \left|f_t\right)\right) \leq \alpha \\ & \quad \Leftrightarrow N\left(lo\,g\left(1 + \frac{L}{h(t)}\right)^+ - (rB + S_t\mu + \frac{1}{2}\ S_t\sigma^2)\tau\right) \end{aligned}$$

$$\Rightarrow L \leq h(t)\left[exp\left(\left(rB(t)+S_t\mu+\tfrac{1}{2}S_t\sigma^2\right)\tau+N^{-1}(\alpha)S_t\sigma\sqrt{\tau}\right)-1\right]^+,$$

which implies

$$Q_t^{\alpha,S} = h_t^S\left[exp\left(\left(rB(t)+S_t\mu+\tfrac{1}{2}S_t\sigma^2\right)\tau+N^{-1}(\alpha)S_t\sigma\sqrt{\tau}\right)-1\right]^+ .$$

Therefore,

$$VaR_t^{\alpha,S} = Q_t^{\alpha,S} = h_t^S\left[1-exp\left(\left(rB(t)+S_t\mu+\tfrac{1}{2}S(t)\sigma^2\right)\tau+N^{-1}(\alpha)S(t)\sigma\sqrt{\tau}\right)\right]^+ .$$

Similarly,

$$E\left[\left(h_t^S-\hbar_{t+\tau}(h_t^S,S_t)\right)_{\left\{\left(h_t^S-\hbar_{t+\tau}(h_t^S,S_t)\right)\right\}\geq -Q^{\alpha,S}(t)}\middle| f_t\right]$$

$$= h_t^S E\left[1-exp\begin{pmatrix}\left(rB(t)+S_t\mu+\tfrac{1}{2}S(t)\sigma^2\right)\tau+S_t\sigma\left(w(t+\tau)-w(t)\right)\\ \times\frac{S_t\sigma\left(w(t+\tau)-w(t)\right)}{S_t\sigma\sqrt{\tau}}\leq N^{-1}(\alpha)\end{pmatrix}\middle| f_t\right]$$

$$= h_t^S\left[\alpha-exp\left(\left(rB(t)+S_t\mu+\tfrac{1}{2}S_t\mu\right)\tau\right)\int_{-\infty}^{N^{-1}(\alpha)}\frac{1}{\sqrt{2\pi}}exp\left(-\frac{\left(x-\sigma\sqrt{S_t\tau}\right)^2}{2}\right)dx\right]$$

$$= h_t^S\left[\alpha-exp\left((rB(t)+S_t\mu)\tau\right)N\left(N^{-1}(\alpha)-\sigma\sqrt{S_t\tau}\right)\right] .$$

In particular,

$$0\leq V_aR_t^{\alpha,S}\leq TCE_t^{\alpha,S}<h_t^S \text{ and } V_aR_t^{\alpha,0}=TCE_t^{\alpha,0}=0.$$

We seek the optimal asset and liability allocation that maximizes (over admissible $\{C_t.L_t\}$) the expected utility of terminal wealth at time T and liability over the entire horizon $[0,T]$, for a risk averse institution that limits its risk by imposing an upper bound on the TCE.

In mathematical terms the stochastic asset-liability control problem with no transaction under a TCE constraint is

$$Max_{(C,L)\in\mathfrak{B}}\,E\left(U(h_t^S)\right), \tag{18}$$

subject to the wealth process

$$\mathrm{h_t^S} = \mathrm{h_0}\exp\int_o^t\left(rB(s)+\mu S(s)+\tfrac{1}{2}\sigma^2S(s)\right)ds+\int_0^t S(s)\sigma dW(s)$$

$$\log\left(\frac{1-\overline{TEC}\,(h_t^S,t)}{h_t^S}\right)-\left(rB(t)+\mu S(t)+\tfrac{1}{2}S(t)\sigma^2\right)\tau+N\left(N^{-1}(\alpha)\right)S(t)\sigma\sqrt{\tau}\ \leq 0,$$

and the TCE constraint for fixed $\Delta t>0$ given by

$$TCE_t^S\leq\rho(h.t),\forall t\epsilon\,[0,\tau] \tag{19}$$

where

$$\rho(S_t,t) = 1 - exp\left(\left(rB(t) + \mu S(t) + \frac{1}{2}S(t)\sigma^2\right)\tau + N\left(N^{-1}(\alpha)\right)S(t)\sigma\sqrt{\tau}\right)$$

and

$$\hat{\rho}(S_t,t) = 1 - exp\big((rB(t) + S_t\mu)\tau\big)\frac{N\left(N^{-1}(\hat{\alpha}) - \sigma\sqrt{S_t\tau}\right)}{\hat{\alpha}}.$$

With probability $1-q$ and $\lambda = 0$, we have $h(t) = B(t)S(t)$ or $S(t) = \frac{h(t)}{B(t)}$. This is based on the classical function which implies that price $S(t)$ of the risky asset equals the ratio of the wealth process $h(t)$ to the price of bond $B(t)$.

Applying the TCE constraint while maximizing the institution's logarithmic utility over asset-liabilities throughout the investment horizon and over the terminal wealth, we have;

$$Max_{(C,L)\in\mathfrak{B}}\, E\left(U(h_t^S)\right),$$

subject to the wealth process

$$\mathrm{h_t^S} = \mathrm{h_0}\exp\int_o^t \left(rB(s) + \mu\frac{h(s)}{B(s)} + \frac{1}{2}\sigma^2\frac{h(s)}{B(s)}\right)ds + \int_0^t \frac{h(s)}{B(s)}\sigma dW(s)$$

$$\log\left(1 - \alpha - \frac{\overline{TEC}\,(h_t^S,t)}{B(t)S(t)}\right) - \left(rB(t) + \mu\frac{h(t)}{B(t)} + \frac{1}{2}\frac{h(t)}{B(t)}\sigma^2\right)\tau + \log N\left(N^{-1}(\alpha)\right)\frac{h(t)}{B(t)}\sigma\sqrt{\tau} \le 0 \quad (20)$$

3 The optimal price

Let $z_t = C(t) - L(t)$ be the value of the net assets at the end of period t (savings). Consider the institution's economy at time t with function over the net assets given by Cochrane (2001):

$$V(z_t, z_{t+1}) = V(z_t) + E_t[V(z_{t+1})]\,, \quad (21)$$

where E_t is the conditional expectation operator over future states at time $t+1$. If we consider the liability factor ϑ and a measure of the institution's impatience to invest, β, (that is β is the relative risk premium coefficient),we may write

$$E_t[V(z_{t+1})] = \vartheta + \beta E_t[V(z_t, z_{t+1})]. \quad (22)$$

Equation (21) now becomes:

$$V(z_t, z_{t+1}) = \frac{\vartheta + V(z_t)}{1-\beta} \quad (23)$$

It has been shown in Osu et al. (2011) that when expressed in terms of cumulative distribution of $z_i(t)$, (23) becomes

$$U(z) = \frac{z^{-(1+\delta)}}{1-\beta}. \quad (24)$$

$$U(z) = \frac{z^{-\gamma}}{1-\beta} \text{ for } z \geq 0.$$

Equation (24) is a power law distribution of aggregate cash flow between an institution and its propensity to invest. It implies a relation between the rank of an institution in the wealth hierarchy and its wealth. Figures 1 and 2 in the appendix show an institution's investment policy with or without transaction cost.

Put $\gamma = (1 + \delta)$ an arbitrary parameter $\delta > 0$ and define the power utility function

$$U(z) = \frac{z^{-(1+\delta)}}{1-\beta} \text{ for } z \geq 0.$$

The parameter β is called the relative risk premium coefficient. The objective of the institution is to choose an allocation of his wealth so as to maximize the expected utility of his terminal wealth, i.e.,

$$V(t, x) = \sup_{S\in\Re_+^2} E_{t,h}[U(x)].$$

The HJB equation associated with this problem is

$$\frac{\partial w}{\partial t}(t, x) + \sup_{S\in\Re_+^2} L^x w(t, x) = 0, \tag{25}$$

Where L^u is the second order linear operator:

$$L^u w(t, x) = (r + (\mu - r)n)h \frac{\partial w}{\partial h}(t, x) + \frac{1}{2}\sigma^2\mu^2 h^2 \frac{\partial^2 w}{\partial h^2}(t, x)$$

From (25) we see that

$$E_{\tau,x}[U(X_\tau)] = \frac{x^{-\gamma}}{1-\beta} E_{t,1}[u(X_\tau)] \text{ and } V(t, x) = \frac{x^{-\gamma}}{1-\beta} V(t, 1).$$

Set $h(t) = V(t, 1)$, and plug the above separability property of V in (25) to get

$$0 = h' - \gamma \sup_{S\in\Re_+^2} \left[r + (\mu - r)v + \frac{1}{2}\sigma^2\mu^2(r + 1)\right] , \tag{26}$$

So that

$$h' = (1 + \delta)h\left[r + \frac{(r-\mu)^2}{2(\delta+2)\sigma^2}\right] \tag{27}$$

Where the maximizer is

$$\hat{u} = \frac{r-\mu}{(\delta+2)\sigma^2} \tag{28}$$

Since $V(T,.) = U(z)$, we seek for a function h satisfying (28) the differential equation together with boundary condition $h(T) = 1$. This allows us to select a unique candidate for the function h

thus;

$$h(t) := e^{b\tau} \tag{29}$$

with

$$b := (1+\delta)\left[r + \frac{(r-\mu)^2}{(\delta+2)\sigma^2}\right], \tau = T - t. \quad (30)$$

Therefore, the function $(t,z) \mapsto \frac{z^{-(1+\alpha)}}{1-\beta} h(t)$ is a classical solution of the HJB .

We now specialize our model by assuming that $u(h) = \frac{h^{-(1+\delta)}}{1-\beta}$ for some $\beta > 0$. In the absence of a TCE constraint, we have;

$$V(h,t) = \frac{h^{-(1+\delta)}}{1-\beta} e^{b\tau}, \quad (31)$$

where b is as in (30) and

$$\hat{S}(h,t) = \frac{r-\mu}{(\delta+2)\sigma^2}. \quad (32)$$

Using (11) and (30), the terminal wealth h_t^S of an institution is in this case log normally distributed as

$$h_t^S = h_0 e^{\left((1+\delta)\left[r + \frac{(r-\mu)^2}{(\delta+2)\sigma^2}\right]\right)\tau}, \quad (33)$$

with mean

$$h_0 e^{\left(r + \frac{(r-\mu)^2}{(\delta+2)\sigma^2}\right)\tau} \quad (34)$$

and standard deviation

$$h_0 e^{\left(r + \frac{(r-\mu)^2}{(\delta+2)\sigma^2}\right)\tau} \sqrt{e^{\frac{(r-\mu)^2}{(\delta+2)\sigma^2}} - 1}. \quad (35)$$

4 Conclusion

The $U(t,z) = \frac{z^{-(1+\delta)}}{1-\beta} h(t)$, and the optimal asset-liability control allocation policy is given by the constant process as in (32).

The $\hat{S}(h,t)$ here represents the price (the value) of the institution asset or liability depending whether $r > \mu$, $r < \mu$ or $r = \mu$.

Given an upper and a lower bound on the fraction $S(t)$, the price allocation of the asset:

$$S^-(Z,t) \le S(t) \le S^+(Z,t),$$

we can verify using the method in Couco et al. (2008) and Akume et al. (2009, 2010) that $h(t) = S(t)B(t)$ is quadratic and satisfies the upper and lower bound such that

$$S^{\pm}(Z,t) =$$
$$\vartheta\sqrt{\tau} \pm N^{-1}(\alpha) \pm \sqrt{\left(\vartheta\sqrt{\tau} \pm N^{-1}(\alpha)\right)^2 - 2\left(\log\left(1 - \alpha - \frac{\overline{TCE}}{Z}\right) + rB\tau + \theta\tau + \frac{1}{2}N^{-2}(\alpha)\right)}. \quad (36)$$

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APPENDIX

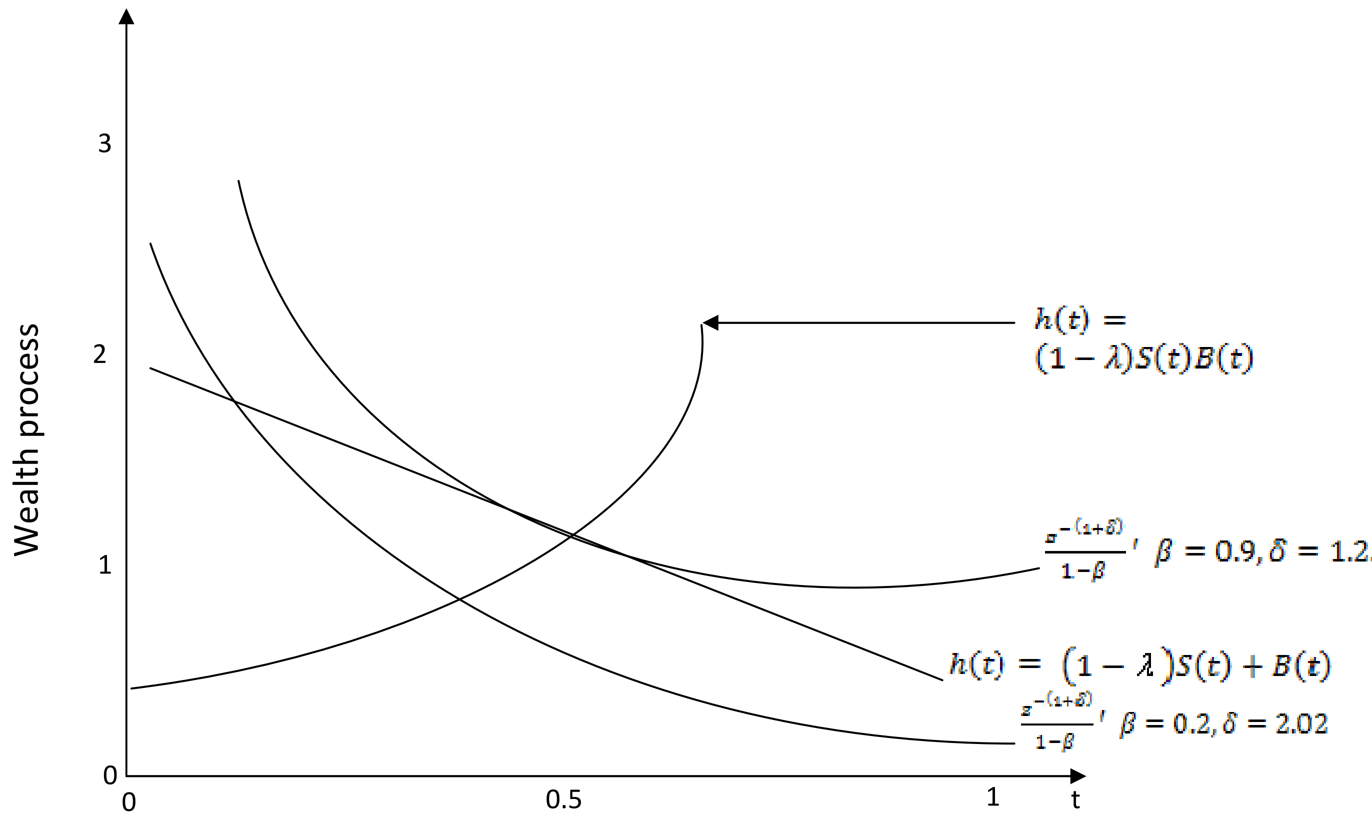

Fig. 1. The institutions Asset – liability over time and the wealth process when $\lambda \neq 0$.The additive wealth decreases with different values of (0 <λ<1) while the multiplicative wealth is greater than zero for $\lambda \neq 1$

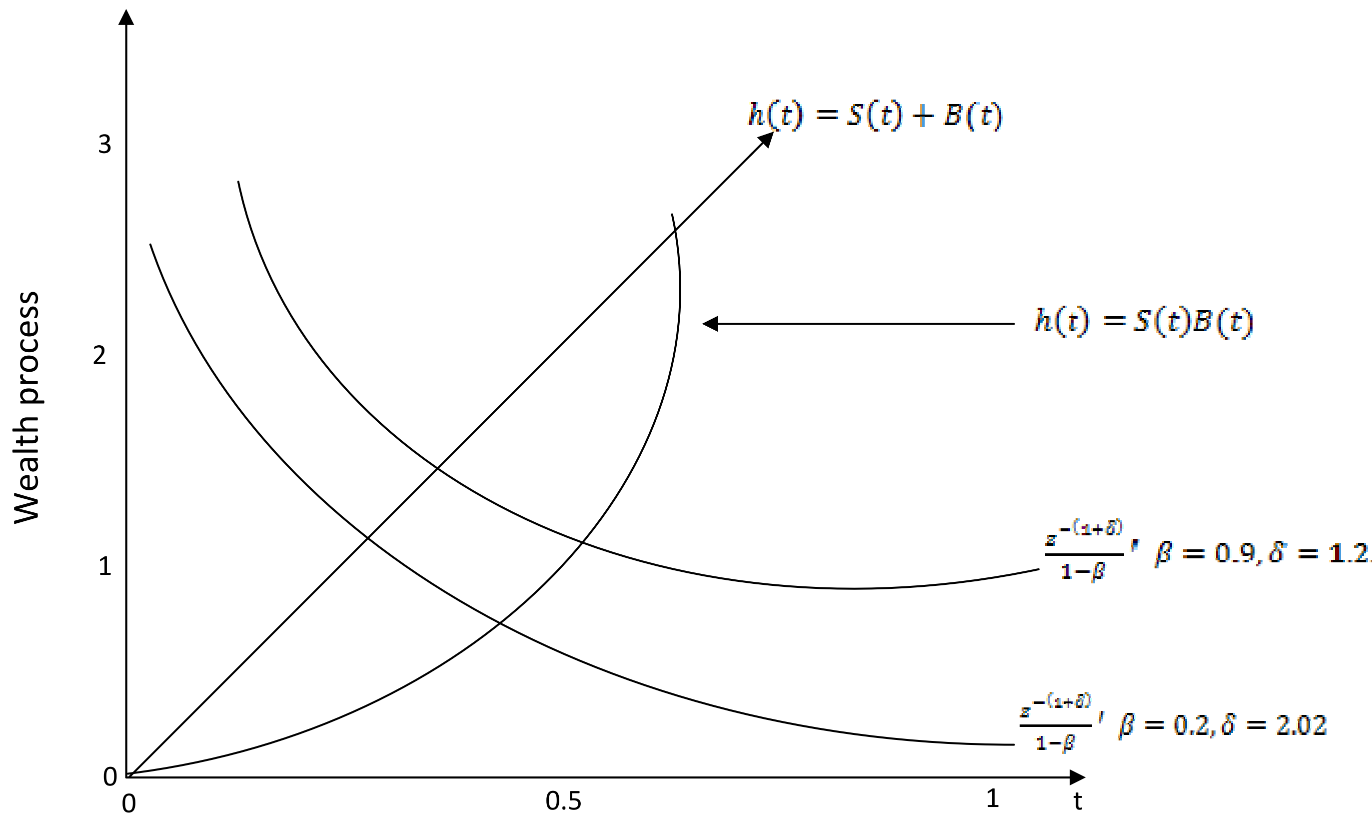

Fig. 2. The institution Asset – liability over time and the wealth process when $\lambda = 0$. The additive wealth increases with time. The multiplicative wealth is zero for $S_0 = B_0 = 0$.