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Twin Paradox and Proper Time

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Abstract

Professors Mohazzbi and Luo [1] published "Despite several attempts have been made to explain the twin paradox … none of the explanations … resolved the paradox. If the paradox can be ever resolved, it requires a much deeper understanding … of the theory of relativity". The deeper understanding of resolving the paradox is by applying more explicit definitions of proper time interval, Lorentz transform, time dilation, and aging time.

Keywords

Twin Paradox, Proper Time, Minkowski Metric, Schwarzschild Metric

1. Introduction

Professors Mohazzbi and Luo documented the failures to resolve the twin paradox [1], to their satisfaction. The following resolves their concerns. Explicit definitions of Lorentz transform, time dilation, proper time and aging time will be given and applied to the twin paradox analysis. The transient/dynamic nature of the Lorentz transform will be introduced. The analysis demonstrates the astronaut returning to Earth ages the same as his twin that stayed on Earth, using constant velocity reference frames in special theory. When the analysis is done using constant acceleration reference frames in general relativity, the result is the same.

2. Characteristics of the Special Relativity Theory Lorentz Transform [1] [2]

The Lorentz transforms frame A, Δt_A, Δx_A coordinates to Δt_B, Δx_B coordinates of frame B, given the velocity (v_{AB}) from frame A to frame B.

Δt time interval units light-seconds.

Δx length interval units light-seconds.

Speed of light $c = 1$.

(Length interval light travels in one light-second)/(time interval light travels in one light-seconds).

When $\Delta x_0 = 0$ and Δt_0, frame 0 has absolute velocity zero (see Equations (4) and (5)).

$$v_{0A} = \Delta x_A / \Delta t_A \quad \text{is absolute velocity frame 0 to frame A.} \tag{1}$$

$$v_{0B} = \nabla x_B / \Delta t_B \quad \text{is absolute velocity frame 0 to frame B.} \tag{2}$$

$$v_{AB} = \left(v_{0B} - v_{0A}\right) / \left(1 - v_{0B} v_{0A}\right) \quad \text{is Lorentz velocities} \tag{3}$$

3. Characteristics of Minkowski Metric [2] [3]

The Minkowski metric relates proper time interval ($\Delta\tau$) [2] [3] to the Lorentz coordinates transformed.

$$\Delta\tau^2 = \Delta t_0^2 = \Delta t_n^2 - \Delta x_n^2, \quad n = 1, 2, 3, \cdots \tag{4}$$

Δt_n and Δx_n are frame n coordinates where the velocity from frame 0 to frame n is

$$v_{0n} = \Delta x_n / \Delta t_n \quad \text{(There are many } v_{0n} \text{ values representing the same } \Delta\tau \text{ value.)} \tag{5}$$

When $\Delta x_0 = 0$ and Δt_0, frame 0 has absolute velocity zero. By inspection of (4), a frame at rest must have its $\Delta x = 0$. Another way to tell if a frame is at rest in space, is an object in the frame has no kinetic energy, it is at rest mass.

Δt_0 ($\Delta\tau$ equals Δt_0) is an invariant of the Lorentz transform. A given a value of $\Delta\tau_g$ restricts the transformed coordinates allowed in a transformed Lorentz reference frame. When coordinates are transformed from frame A to frame B using any constant velocity between the frames, the coordinates (Δt_A and Δx_A) and coordinates (Δt_B and Δt_B) are restricted by the Minkowski metric.

$$\Delta\tau_g^2 = \Delta t_A^2 - \Delta x_A^2 = \Delta t_B^2 - \Delta x_B^2 \tag{6}$$

Proper time interval is defined as the Δt_0 (clock time interval between two events) observed in your frame (0) when the two events happen in frame (0). If the two events did not happen in your frame (0), Δt_0 can be established in your frame (0), Event 1 flashes a green light, Event 2 flashed a red light, and the distances to the two events is known. If the light sources are moving there will be a frequency shift of the red and green flashes.

Consideration of the transit/dynamic nature of the Lorentz transform: consider two rest frames A (Δx_A equals 0) and B (Δx_B equals 0) that are Δx_D distance from frame A to frame B. Let a clock move from frame A to frame B at a constant velocity v (Event 1), stops (Event 2), and return to frame A at a velocity—v (Event 3). Upon return to frame A, it stops (Event 4).

Start

$\Delta t_A = 0$	$\Delta x_A = 0$	$\Delta t_B = 0$	$\Delta x_B = 0$

Event 1

$\Delta t_A = \Delta x_D / v$	$\Delta x_A = 0$	$\Delta t_B = \left(\Delta x_D / v\right) / \sqrt{1 - v^2}$	$\Delta x_B = \Delta x_D$

Event 2

$\Delta t_A = \Delta x_D / v$	$\Delta x_A = 0$	$\Delta t_B = \Delta x_D / v$	$\Delta x_B = 0$

Event 3

$\Delta t_A = \left(\left(-\Delta x_D\right)/-v\right)\Big/\sqrt{1-\left(-v\right)^2}$ $\quad \Delta x_A = -\Delta x_D \quad \Delta t_B = -\Delta x_D/-v \quad \Delta x_B = 0$

$+\Delta x_D/v \qquad +\Delta x_D \qquad +\Delta x_D/v$

Event 4 $\Delta t_A = 2\Delta x_D/v \qquad \Delta x_A = 0 \qquad \Delta t_B = 2\Delta x_D/v \qquad \Delta x_B = 0$

Total proper time = Event 1 ($\Delta x_D/v$) + E 2 (0) + E 3 ($\Delta x_D/v$) + E 4 (0) = $2\Delta x_D/v$.

This illustrates, snap shots of coordinates as the clock moves from one stationary Lorentz reference to another and then returns. The key considerations are Lorentz frames, standing still in space with their Δx equal to zero. The going out $\Delta\tau$ equals the return $\Delta\tau$. The proper time interval $\Delta\tau$ (between Start and Event 4) is $2\Delta x_D/v$. Time dilation of an interval is observable only for an instant.

Consider the relationship between proper time interval $\Delta\tau$ and aging interval. An example of an aging interval is the time between the birth (Event 1) and death of a day fly (Event 2), which is represented by $\Delta\tau$ = 24 hours, which is the life span of a day fly. Proper time interval ($\Delta\tau$) can represent the aging interval, which is an interval between two events.

This all means astronaut returning to Earth ages the same as his twin that stayed on Earth.

4. Characteristics of General Relativity Schwarzschild Metric [2] [3] [4]

The Schwarzschild Metric models an astronaut accelerating away from Earth and then revering his acceleration, returning to Earth. It is assumed the acceleration and velocity in no way affects his heath. The version of the metric is:

$$\Delta\tau^2 = \alpha_F \Delta t_n^2 - \alpha_F^{-1}\Delta x_n^2 . \tag{7}$$

n identifies the reference frame. $n = 1, 2, 3, \cdots$.

Δt_n is time interval in frame n.

Δx_n is length interval in frame n.

α_F is parameter representing the constant gravity force moving an object Δx_n distance in Δt_n time. α_F can have different values in a different frame.

(α = 1, represents no force and the Schwarzschild metric becomes the Minkowski metric.)

(α = 0, represents infinite force.)

$\Delta\tau$, proper time interval, is invariant of Δt_k and Δx_k coordinates in frame (k) where α_F can have different values in different frames representing a different gravity force in that frame.

Consider astronaut rockets from Earth at constant acceleration α_A for a proper time interval of 1/4 ($\Delta\tau_T$) and reveres his acceleration for 1/4 ($\Delta\tau_T$) returning to zero velocity. Then keeping acceleration toward Earth for 1/4($\Delta\tau_T$) and again revering his acceleration away from Earth for 1/4($\Delta\tau_T$), returning to Earth with zero velocity. The total proper time interval ($\Delta\tau_T$), experienced by the astronaut is $\Delta\tau_T$. His twin has stayed on Earth for the same proper time interval ($\Delta\tau_T$). Both twins will have aged at the same proper time interval.

5. Conclusion

The astronaut returning to Earth ages the same as his twin that stayed on Earth. This analysis did not address the effects of mass changes caused by an object's absolute velocity or acceleration.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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