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Research Article

Control of the Spread of COVID-19 by the Sentinel Method and Numerical Simulation of the Studied Model Solution

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The COVID-19 (coronavirus disease) pandemic represents a global public health emergency unprecedented in recent history. This explains the growing interest of scientists in this subject. Indeed, the question of the COVID-19 pandemic has led to numerous scientific works, with the aim of estimating the reproduction number, the start date of the epidemic or the cumulative incidence. Their results have contributed to epidemiological surveillance and informed public health policy decisions. In our work, using basic data and statistics from the city of Ouagadougou in Burkina Faso, we first consider a mathematical model of COVID-19 transmission taking into account the possibility of transmission from dead populations to susceptible populations; then, we use another method which is the sentinel's method of JL Lions to estimate the number of infected populations without any time trying to know the beginning of the epidemic; finally, we highlight the numerical simulation of the considered model solution.

1. Introduction

In practice, this study is taking place within the framework of the fight against COVID-19. The COVID-19 (coronavirus disease 2019) pandemic is a global public health problem, the first cases of which were first reported in Wuhan, China, on 31st December 2019. Our work is important in the strategy fight against COVID-19. COVID-19 is a respiratory infection caused by a Coronovirus called SARS-Cov6-2. A first strategy to control the spread of COVID-19 in the world consisted in confining the population, this made it possible to reduce the number of reproduction. In the theoretical framework, to estimate the number of reproduction, few authors have used mathematical models with spatial diffusion. However, to predict the evolution of the epidemic, researchers have used mathematical models of propagation such as the SIR epidemiological model (see, for example, [1]). In our work, we considered a SCIRD-type model of COVID-19 transmission in which we include mortality terms and spatial diffusion. Indeed, let us assume the most realistic situation of geographical spread of epidemics; moreover, we will consider the case of disturbances and incomplete data. The prediction of unknown data with the confrontation of real parameters is sometimes necessary, and our objective is to show that by using the sentinel method of JL Lions, a mathematical answer can nevertheless be given. More precisely, let Ω be an open and bounded domain of $\mathbb{R}^N$. For the time $T > 0$, set $Q = (0, T) \times \Omega$, $\Sigma =$

$(0, T) \times \Gamma$. We denote by ν the outer normal on Γ. Then, consider the following problem:

$$\begin{cases} \dfrac{\partial S}{\partial t} - \gamma \Delta S = \Lambda - \dfrac{\varepsilon SD}{N} - (\delta(x) + \mu)S + \eta R, & \text{in } Q, \\ \dfrac{\partial C}{\partial t} - \gamma \Delta C = \delta(x)\theta S + \dfrac{\varepsilon SD}{N} - (\beta + \mu + \pi)C, & \text{in } Q, \\ \dfrac{\partial I}{\partial t} - \gamma \Delta I = \delta(x)(1-\theta)S + \pi C - (\alpha + \mu + \sigma)I + \lambda \widehat{I}, & \text{in } Q, \\ \dfrac{\partial R}{\partial t} - \gamma \Delta R = \beta C + \alpha I - (\mu + \eta)R, & \text{in } Q, \\ \dfrac{\partial D}{\partial t} - \gamma \Delta D = \sigma I, & \text{in } Q, \\ S(0) = S^0 + \tau \widehat{S}^0, & \text{in } \Omega, \\ C(0) = C^0, & \text{in } \Omega, \\ I(0) = I^0, & \text{in } \Omega, \\ R(0) = R^0, & \text{in } \Omega, \\ D(0) = D^0, & \text{in } \Omega, \\ S = 0, & \text{on } \Sigma, \\ C = 0, & \text{on } \Sigma, \\ I = 0, & \text{on } \Sigma, \\ R = 0, & \text{on } \Sigma, \\ D = 0, & \text{on } \Sigma. \end{cases} \quad (1)$$

The function $S(t, x)$ represents susceptible persons at risk of contacting COVID-19 at time t at the point $x \in \Omega$, the function $C(t, x)$ represents carriers (dead corpse) that transmit the COVID-19 at time t at the point $x \in \Omega$, the function $I(t, x)$ describes infective persons capable of transmitting the COVID-19 to persons at risk at time t at the point $x \in \Omega$, the function $R(t, x)$ represents recovered persons who have been treated of COVID-19, and the function $D(t, x)$ gives the total number of deaths at time t at the point $x \in \Omega$.

It should be noted that this type of mathematical model without spatial diffusion has already been studied by Atangana [2], it uses Italian data to propose a mathematical model of COVID-19 transmission. The COVID-19 problem have been studied by several authors. In [3], Tuite et al. used an age-structured compartmental model to analyze the transmission of COVID-19 in the population of Ontario, Canada. In [4], Zhao et al. modeled the epidemic curve of the 2019-ncov case time series in China while accounting for the impact of variations in disease reporting rate. In [5], Wang et al. estimated the time-varying reproduction numbers in China, using the revolving equation determined by the serial interval (SI) of COVID-19. For the model (1), we have

$$\delta(x) = k e^{-|x|} p \left(\frac{I + wC}{N} \right), \quad k = \begin{cases} 0, & \text{if } |x| > 1, \\ >0, & \text{if } |x| < 1. \end{cases} \quad (2)$$

The parameters of the considered model are presented in Table 1.

In a biological explanation of the model, let N be the density of the total population $N =: S + C + I + R + D$. As there is a term of spatial diffusion $-\gamma \Delta S$, the total population N is not constant, and therefore, $\partial_t N \neq 0$.

The term $\lambda \widehat{I}$ represents the disturbance around the infected population whose density is not completely known; $\tau \widehat{S}^0$ is the missing term of the population likely to be infected. We suppose that $\|\widehat{I}\|_{L^2(Q)} \leq 1$ and also that $\|\widehat{S}^0\|_{L^2(\Omega)} \leq 1$. It is the real parameters λ and τ which are unknown; the other parameters are known. If we add the first five equations of the model (1) we are studying, we get the simplified system:

$$\begin{cases} \partial_t N - \gamma \Delta N + \mu N = \Lambda + \lambda \widehat{I}, & \text{in } Q, \\ N(0) = N^0 + \tau \widehat{S}^0, & \text{on } \Omega, \\ N = 0, & \text{on } \Sigma, \end{cases} \quad (3)$$

where $N^0 =: S^0 + C^0 + I^0 + R^0 + D^0$. Therefore, in the following, system (3) is considered and we assume the following:

(H1): $\mu \in C([0, T] \times \bar{\Omega}) \quad \mu(t, x) \geq 0$ a.e in Q.

(H2): $\nabla \mu \in [L^\infty(Q)]^{n.}$

Remark 1. It is assumed that the initial data S^0, C^0, I^0, R^0, and D^0 are known function ant belong to $L^2(\Omega)$. Under the assumptions (H1) and (H2), according to Lions and Magenes [6], system (3) of parabolic type admits a unique solution such that

$$N \in \mathscr{C}^0\left([0, T[\,; L^2(\Omega)\right) \cap \mathscr{C}^0\left(]0, T[\,; H^2(\Omega) \cap H_0^1(\Omega)\right). \quad (4)$$

Now, the problem is as follows: is there a method to get information about the term $\lambda \widehat{I}$ (so-called pollution term) from the infected population, insensitive to the missing term from the susceptible population $\tau \widehat{S}^0$ (so-called perturbation term)? In other words, we are interested in identifying the unknown parameters in model (3) which has incomplete data; more precisely, we are interested in the identification of pollution in the COVID-19 phenomenon, in the sense that we know little about the initial data as well as certain missing terms. The position (internal) and the nature of pollution (COVID-19) are known, but we do not know its amplitude (number of people infected by COVID-19). Our goal is to find a method to obtain information on these amplitudes.

A partial answer can be obtained using the least squares method which is to take the unknowns, $\{\lambda \widehat{I}, \tau \widehat{S}^0\} = \{w, \nu\}$ as control variables, and we can not clearly separate ν and w. The sentinel method of Lions [2] provides the right answer to this type of problem. In this paper, we construct sentinels when the supports of the observation function and of the control function are included in two different open subsets of $\mathbb{R}^N$ (see Nakoulima [7]).

Table 1: Parameters of the considered model.

Symbol	Interpretation
μ	Rate of natural death
Λ	Recruitment rate into $S(t, x)$
θ	Probability of an $S(t, x)$ class to join $C(t, x)$ class
σ	Death rate induced by COVID-19
β	Recovery rate of $C(t, x)$ class
$\delta(x)$	Force of infection of class $S(t, x)$
α	Recovery rate of $I(t, x)$ class
π	Rate of which an $C(t, x)$ class is recovered
η	Rate of which treated persons become $C(t, x)$ class
γ	Disturbance parameter
ε	Rate of infectivity between $S(x, t)$ class and $D(x, t)$ class
p	Rate of proportion that a contact is efficient enough to cause infection
w	Transmission parameter for $C(x, t)$ class
k	Rate of contact

Several authors studied the sentinel problem. We refer to [7–9] and the references therein. In [10], Ainseba et al. used the method of sentinels to identify parameters of pollution in a river. Bodart and Demeestere applied it in [11] to identify an unknown boundary. In [12], Soma and Sawadogo studied the boundary sentinels with given sensitivity in population dynamics problem. In [9], Mophou and Nakoulima studied the problem of sentinels with given sensitivity. Sawadogo introduced in [13] the distributed sentinels into the equation of the dynamics of populations to study a population subject to a migratory phenomenon. Recently in [14], Hamadi et al. have studied the existence of construction of a sentinel for the epidemiological model SIR with spatial diffusion. In this paper, we apply the sentinel method to identify parameter in a mathematical model of COVID-19 transmission with spatial diffusion and incomplete data which takes into account the possibility of transmission from dead populations to susceptible populations. The problem is as follows: given $h_0 \in L^2(U \times O)$, find a control w in $L^2(U \times \omega)$ such that if $N = N(\lambda, \tau)$ is solution of (3) and S' is defined by (6); then (7) and (8) hold.

The remainder of this paper is as follows: in Section 2, we briefly explain the concept of the sentinel In Section 3, we establish the equivalence between sentinel problem and null controllability problem. Sections 4 and 5 are devoted, respectively, to preliminary results and proof of main result. In Section 6, we formulate the information given by sentinel. In Section 7, we highlight the numerical simulation of the considered model solution.

2. Sentinel Method

The sentinel concept relies on the following three objects: some state equation (for instance), some observation function, and some control function w to be determined.

A state equation represented here by (3) and we denote by $N = N(t, x, \lambda, \tau) = N(\lambda, \tau)$ depends on two parameters λ and τ the unique solution of (3).

An observation on nonempty open subset $O \subset \Omega$ is called the observatory set. The observation is N in O, for the time T. We denote by N_{obs} this observation

$$N_{\text{obs}} = m_0 \in L^2((0, T) \times O). \tag{5}$$

A function $S' = S'(\lambda, \tau)$ called "sentinel". Let $h_0 \in L^2((0, T) \times O)$. Let on the other hand ω be some open and nonempty subset of Ω such that $\omega \neq O$.

For a control function $\mathrm{w} \in L^2((0, T) \times \omega)$, we define the functional

$$S'(\lambda, \tau) = \int_0^T \int_O h_0 N(\lambda, \tau) dt da dx + \int_0^T \int_\omega \mathrm{w} N(\lambda, \tau) dt da dx. \tag{6}$$

We say that S' defines a sentinel for the problem (1) if there exists w such that S' is insensitive (at first order) with respect the to missing terms $\tau \widehat{N}^0$, which means

$$\frac{\partial S'}{\partial \tau}(0, 0) = 0, \quad \forall \widehat{N}^0, \tag{7}$$

where here $(0; 0)$ corresponds to $\lambda = \tau = 0$ and w is of minimal norm in $L^2((0, T) \times \omega)$. That is,

$$\|\mathrm{w}\|_{L^2(0,T)\times\omega} = \min_{u \in L^2((0,T)\times\omega)} \|u\|_{L^2((0,T)\times\omega)}. \tag{8}$$

3. Null Controllability Problem

We show in this section that the existence of the sentinel comes to null controllability property. We begin by transforming the insensibility condition (7).

Set

$$N_\tau = \frac{d}{d\tau} N(\lambda, \tau)\Big|_{\lambda=\tau=0}. \tag{9}$$

Then, the function $N_\tau = S_\tau + C_\tau + I_\tau + R_\tau + D_\tau$ is solution of

$$\begin{cases} \partial_t N_\tau - \gamma \Delta N_\tau + \mu N_\tau = 0, & \text{in } Q, \\ N_\tau(0) = \widehat{S}^0, & \text{in } \Omega, \\ N_\tau = 0, & \text{on } \Sigma. \end{cases} \tag{10}$$

Problem (10) is linear and has a unique solution N_τ. The insensibility condition (7) holds if and only if

$$\int_Q (h_0 \chi_O + \mathrm{w} \chi_\omega) N_\tau dt da dx = 0. \tag{11}$$

We can transform (11) by introducing the classical adjoint state. More precisely, we define the function $q = q(t\,;x)$ as the solution of the backward problem:

$$\begin{cases} -\partial_t q - \gamma\Delta q + \mu q = h_0\chi_O + \mathrm{w}\chi_\omega, & \text{in } Q, \\ q(T) = 0, & \text{in } \Omega, \\ q = 0, & \text{on } \Sigma. \end{cases} \quad (12)$$

Since $h_0\,; \in L^2((0,T)\times O)$ and $\mathrm{w} \in L^2((0,T)\times\omega)$, the adjoint problem admits a unique solution given by $q \in L^2(Q) \cap C([0,T]\,;H^{-1}(\Omega))$. The function q depends on the control w that we shall determine:

Indeed, if we multiply the first equation in (12) by N_τ and we integrate by parts over Q, we obtain

$$\int_Q (h_0\chi_O + \mathrm{w}\chi_\omega)N_\tau dtdx = \int_\Omega q(0)\widehat{S}^0 dx, \quad \forall \widehat{S}^0 \in L^2(\Omega). \quad (13)$$

So, condition (7) (or (11)) is equivalent to

$$q(0) = 0, \quad \text{in } \Omega. \quad (14)$$

Thus, sentinel problems (6), (7), and (8) are equivalent to the following null controllability problem: given $h_0 \in L^2((0,T)\times O)$, find a control w in $L^2((0,T)\times\omega)$ such that if q is the solution of (12), then (8) and (14) hold.

In the following, we set

$$\begin{cases} L = \partial_t - \gamma\Delta + \mu id, \\ L^* = -\partial_t - \gamma\Delta + \mu id, \\ \mathscr{V} = \left\{\rho \in C^\infty(\bar{Q}), \rho_{|_\Sigma} = 0\right\}. \end{cases} \quad (15)$$

4. Existence of a Sentinel

For the study of the existence of a sentinel, we use a method developed in [15].

We begin with some observability inequality. Using (15), we have the following:

Proposition 2. *Let be $\rho \in \mathscr{V}$; then, there exists a positive constant $C = C(\Omega, \omega, O, T)$ such that*

$$\int_Q \frac{1}{\theta'^2}|\rho|^2 dtdadx \le C\left[\int_Q |L\rho|^2 dtdadx + \int_0^T\int_\omega |\rho|^2 dtdadx\right], \quad (16)$$

where $\theta' \in C^2(Q)$ positive with $1/\theta'$ bounded

Proof. See Fursikov and Imanualov [16]. □

According to the RHS of (16), we consider the space $\mathscr{V}$ endowed with the bilinear form $a(.;.)$ defined by

$$a(\rho\,;\widehat{\rho}) = \int_Q L\rho L\widehat{\rho}\, dtdadx + \int_0^T\int_\omega \rho\widehat{\rho}\, dtdadx. \quad (17)$$

According to Proposition 2, this symmetric bilinear form is a scalar product on $\mathscr{V}$.

Let V be the completion of $\mathscr{V}$ with respect to the norm

$$\widehat{\rho} \mapsto \|\widehat{\rho}\|_V = \sqrt{a(\rho\,;\widehat{\rho})}. \quad (18)$$

Then, V is a Hilbert space for the scalar product $a(\rho\,;\widehat{\rho})$ and the associated norm.

Remark 3. We can precise the structure of the elements of V. Indeed, let $H_{\theta'}(Q)$ be the weigthed Hilbert space defined by

$$H_{\theta'}(Q) = \left\{\rho \in L^2(Q) \text{ such that} \int_Q \frac{1}{\theta'^2}|\rho|^2 dtdadx < \infty\right\}, \quad (19)$$

endowed with the natural norm $\|\rho\|_\theta = (\int_Q (1/\theta'^2)|\rho|^2 dtdadx)^{1/2}$. This shows that V is imbedded continuously in $H_{\theta'}(Q)$: $\|\rho\|_\theta \le C\|\rho\|_V$.

Now if $h_0 \in L^2(Q)$ and $\theta' h_0 \in L^2(Q)$(i.e.,$h_0 \in L^2_{\theta'}(Q)$), then from (16) and the Cauchy-Schwartz inequality, we deduce that the linear form defined on V by

$$\rho \mapsto \int_Q h_0\chi_O\rho\, dtdadx \quad (20)$$

is continuous. Therefore, from the Lax-Milgram theorem, there exists a unique $u \in V$ solution of the variational equation:

$$\mathrm{a}(\rho\,;\widehat{\rho}) = \int_Q h_0\chi_O\widehat{\rho}\, dtdadx, \quad \forall\widehat{\rho} \in V. \quad (21)$$

Proposition 4. *Assume that $h_0 \in L^2_{\theta'}(Q)$, and let ρ be the unique solution of (21). We set*

$$w = -\rho\chi_\omega \quad (22)$$

and

$$q = L\rho. \quad (23)$$

Then, the pair $(\mathrm{w}\,;q)$ is such that (12)–(14) hold (i.e., there is some insensitive sentinel defined by (6) and (7)).

Moreover, we have

$$\begin{cases} \|\rho\|_V \leq C\|\theta' h_0\chi_O\|_{L^2(Q)}, \\ \|w\|_{L^2(Q_\omega)} \leq C\|\theta' h_0\chi_O\|_{L^2(Q)}, \\ \|q\|_{L^2(Q)} \leq C\|\theta' h_0\chi_O\|_{L^2(Q)}, \end{cases} \tag{24}$$

where C is a constante and $L^2(Q_\omega) = (0, T) \times \omega$.

5. Construction of the Sentinel

Having shown the existence of a sentinel, our goal now is to build this sentinel for the (1). Note that the existence of this sentinel justifies the existence of the optimal control.

5.1. Existence of the Optimal Control. For the following, we will consider the following optimization problem:

$$(\mathrm{P}):\ \min_{(\mathrm{w},q)\in E} \frac{1}{2}\|\mathrm{w}\|,$$

$$E = \left\{ (w,q) \begin{cases} -\partial_t q - \gamma\Delta q + \mu q = h_0\chi_O + w\chi_\omega, & \text{in } Q \\ q(0) = q(T) = 0, & \text{in } \Omega \\ q = 0, & \text{on } \Sigma \end{cases} \right\}. \tag{25}$$

Theorem 5. *There is a unique couple $(\widehat{w}, \widehat{q})$ solution of the problem (P).*

Proof. By Proposition 4, the domain E is nonempty. On the other hand, the map $w \mapsto \|w\|_{L^2(Q_\omega)}$ is continuous, coercive, and strictly convex; then, we deduce that there is a unique solution for the problem (P) □

5.2. Penalization Method. In this subsection, we are concerned with the optimality system for $(\widehat{\mathrm{w}}, \widehat{q})$. Since a classical way to derive this optimality system is the method of penalization due to Lions [8], here, we use this method. For this, we introduce the penalized cost function:

$$J_\varepsilon(w,z) = \frac{1}{2}\left\| w|^2_{L^2(Q_\omega)} + \frac{1}{2\varepsilon}\|L^* z - h_0\chi_O - w\chi_\omega \right\|^2_{L^2(Q)}, \tag{26}$$

where $\varepsilon > 0$.

We consider the following problem (P_ε):

$$(P_\varepsilon):\ \begin{pmatrix} \min J_\varepsilon(\mathrm{w}, z)_{L^2(Q_\omega)}, \\ (w,z) \in U, \end{pmatrix} \tag{27}$$

with

$$U = \begin{cases} (w,z), & \text{such that } L^* z \in L^2(Q), \\ z(0) = z(T) = 0, & \text{in } \Omega, \\ z = 0, & \text{on } \Sigma. \end{cases} \tag{28}$$

The following proposition gives the existence of the solution for (P_ε).

Proposition 6. *It is assumed that the assumptions of the previous section are satisfied. Then, there is a couple $(w_\varepsilon, z_\varepsilon)$ a as unique solution of the problem (P_ε).*

Proof. Since $E \subset U$ and $E \neq \varnothing$, consequently, U is nonempty and is closed. Moreover, J_ε is continuous, coercive, and strictly convex. Then, the problem (P_ε) has a unique solution denoted by $(w_\varepsilon, z_\varepsilon)$. □

Now, we study the convergence of the couple $(\mathrm{w}_\varepsilon, z_\varepsilon)$ when $\varepsilon \longrightarrow 0$. For this, we have the following result:

Proposition 7. *Let $(w_\varepsilon, z_\varepsilon)$ the unique solution of the problem (P_ε). Then,*

$$\begin{aligned} &w_\varepsilon \rightharpoonup \widehat{w} \text{ weakly in } L^2(Q_\omega), \\ &z_\varepsilon \rightharpoonup \widehat{q} \text{ weakly in } H^{2,1}(Q). \end{aligned} \tag{29}$$

Proof. As $(w_\varepsilon, z_\varepsilon)$ is the solution of (P_ε), then,

$$J_\varepsilon(w_\varepsilon, z_\varepsilon) \leq J_\varepsilon(w,z), \quad \forall (w,z) \in U, \tag{30}$$

especially for $(\widehat{\mathrm{w}}, \widehat{q}) \in E \subset U$. Inequality (30) is written

$$J_\varepsilon(w_\varepsilon, z_\varepsilon) \leq J_\varepsilon(\widehat{w}, \widehat{q}) = \frac{1}{2}\|\widehat{w}\|^2_{L^2(Q_\omega)}, \tag{31}$$

which implies

$$\frac{1}{2}\|w_\varepsilon\|^2_{L^2(Q_\omega)} + \frac{1}{2\varepsilon}\|L^* z_\varepsilon - h_0\chi_O - w\chi_\omega\|^2_{L^2(Q)} \leq \frac{1}{2}\|\widehat{w}\|^2_{L^2(Q_\omega)} = C. \tag{32}$$

Thus,

$$\|w_\varepsilon\|_{L^2(Q_\omega)} \leq C, \tag{33}$$

$$\|L^* z_\varepsilon - h_0\chi_O - w\chi_\omega\|_{L^2(Q)} \leq C\sqrt{\varepsilon}. \tag{34}$$

From (33), we conclude that w_ε is bounded in $L^2(Q_\omega)$. So

$$w_\varepsilon \rightharpoonup w_0 \quad \text{in} \quad L^2(Q_\omega). \tag{35}$$

On the other hand, because we have $(w_\varepsilon, z_\varepsilon) \in U$ and according to the inequality (34), we have

$$\begin{cases} L^* z_\varepsilon = h_0 \chi_O + \mathrm{w}\chi_\omega + h_\varepsilon, \\ z_\varepsilon(0) = z_\varepsilon(T) = 0, & \text{in} \quad \Omega, \\ z_\varepsilon = 0, & \text{on} \quad \Sigma. \end{cases} \tag{36}$$

With

$$h_\varepsilon = L^* z_\varepsilon - h_0 \chi_O - w\chi_\omega. \tag{37}$$

Given z_ε solution of (36), then $z_\varepsilon \in H^{2,1}(Q)$. Moreover, we have

$$\|z_\varepsilon\|_{H^{2,1}(Q)} \le C_1 \|h_0\chi_O + w\chi_\omega + h_\varepsilon\|_{L^2(Q)}. \tag{38}$$

Since $\|h_\varepsilon\|_{L^2(Q)} \le C\sqrt{\varepsilon}$, then,

$$\|z_\varepsilon\|_{H^{2,1}(Q)} \le C, \tag{39}$$

where C is a constant. Therefore, there is a subsequence denoted by $(z_\varepsilon)_\varepsilon$, such that $z_\varepsilon \rightharpoonup z_0$ in $H^{2,1}(Q)$. As the injection of $H^{2,1}(Q)$ into $L^2(Q)$ is compact and by passing to the limit, we finally find

$$\begin{cases} L^* z_0 = h_0 \chi_O + \mathrm{w}\chi_\omega, & \text{in} \quad Q, \\ z_0(T) = 0, & \text{in} \quad \Omega, \\ z_0 = 0, & \text{on} \quad \Sigma. \end{cases} \tag{40}$$

On the other hand, J_ε is convex and continuous; then,

$$\frac{1}{2}\|\mathrm{w}_0\|^2_{L^2(Q_\omega)} = J_\varepsilon(\mathrm{w}_0, z_0) \le \lim \inf_{\varepsilon \longrightarrow 0} J_\varepsilon(\mathrm{w}_\varepsilon, z_\varepsilon). \tag{41}$$

Using in (41), the estimate

$$J_\varepsilon(w_\varepsilon, z_\varepsilon) \le J_\varepsilon(\widehat{w}, \widehat{q}) = \frac{1}{2}\|\widehat{w}\|^2_{L^2(Q_\omega)}. \tag{42}$$

We get

$$\|w_0\|^2_{L^2(Q_\omega)} \le \|\widehat{w}\|^2_{L^2(Q_\omega)}. \tag{43}$$

Finally, as $(\widehat{w}, \widehat{q})$ is the unique solution of (P), then, $\mathrm{w}_0 = \widehat{\mathrm{w}}$. On the other hand z_0 is solution of (40) and by uniqueness of the solution for the heat equation, we deduce that $z_0 = \widehat{q}$. □

The following proposition gives the optimality system for the pair $(\mathrm{w}_\varepsilon, z_\varepsilon)$.

Proposition 8. *The problem (P_ε) admits an optimal solution $(w_\varepsilon, z_\varepsilon)$ if and only if, there exists a unique function $\rho_\varepsilon \in L^2(Q)$ such that $\{w_\varepsilon, z_\varepsilon, \rho_\varepsilon\}$ is the solution of the following optimality system:*

$$\begin{cases} L^* z_\varepsilon = h_0 \chi_O + w_\varepsilon \chi_\omega + \varepsilon\rho_\varepsilon, & in \quad Q, \\ z_\varepsilon(T) = 0, & in \quad \Omega, \\ z_\varepsilon = 0, & on \quad \Sigma, \end{cases} \tag{44}$$

with

$$z_\varepsilon(0) = 0 \quad in \quad \Omega \tag{45}$$

and with

$$\begin{gathered} w_\varepsilon = -\rho_\varepsilon \chi_\omega, \\ \begin{cases} L\rho_\varepsilon = 0, & in \quad Q, \\ \rho_\varepsilon(0) = 0, & in \quad \Omega, \\ \rho_\varepsilon = 0, & on \quad \Sigma. \end{cases} \end{gathered} \tag{46}$$

Proof. Since $(\mathrm{w}_\varepsilon, z_\varepsilon)$ is the unique solution of (P_ε) then, by applying the Euler-Lagrange, optimality conditions, we find

$$\frac{\partial}{\partial\lambda} J_\varepsilon(w_\varepsilon + \lambda w, z_\varepsilon)_\lambda = 0, \quad \forall w \in L^2(Q_\omega), \tag{47}$$

or

$$\begin{cases} \dfrac{\partial}{\partial\lambda} J_\varepsilon(w_\varepsilon, z_\varepsilon + \lambda Z)_\lambda = 0, & \forall z \in C^\infty(\bar{Q}) \text{ such that} \\ z = 0 \text{ on } \Sigma, & z(T) = z(0) = 0 \text{ in } \Omega. \end{cases} \tag{48}$$

From the definition of the functional J_ε and the linearity of the operator L^*, $J_\varepsilon(\mathrm{w}_\varepsilon + \lambda\mathrm{w}, z_\varepsilon)$ is written:

$$\begin{aligned} J_\varepsilon(w_\varepsilon + \lambda w, z_\varepsilon) = {} & \frac{1}{2}\|w_\varepsilon + \lambda w\|^2_{L^2(Q_\omega)} \\ & + \frac{1}{2\varepsilon}\|L^* z - h_0\chi_O - (w_\varepsilon + \lambda w)\chi_\omega\|^2_{L^2(Q)}. \end{aligned} \tag{49}$$

So

$$\begin{aligned} & J_\varepsilon(w_\varepsilon + \lambda w, z_\varepsilon) - J_\varepsilon(w_\varepsilon, z_\varepsilon) \\ & = \frac{1}{2}\lambda^2\|w\|^2_{L^2(Q_\omega)} + \lambda\langle w_\varepsilon, w\rangle_{L^2(Q)} + \frac{1}{2\varepsilon}\lambda^2\|w\chi_\omega\|^2_{L^2(Q)} \\ & \quad + \frac{\lambda}{\varepsilon}\langle w_\varepsilon\chi_\omega, w\chi_\omega\rangle_{L^2(Q)} - \frac{1}{\varepsilon}\langle L^* z_\varepsilon - h_0\chi_O, w_\varepsilon\chi_\omega\rangle_{L^2(Q)} \\ & \quad - \frac{\lambda}{\varepsilon}\langle L^* z_\varepsilon - h_0\chi_O, w\chi_\omega\rangle_{L^2(Q)} - \frac{1}{2\varepsilon}\|w\chi_\omega\|^2_{L^2(Q)} \\ & \quad + \frac{1}{\varepsilon}\langle L^* z_\varepsilon - h_0\chi_O, w_\varepsilon\chi_\omega\rangle_{L^2(Q)}. \end{aligned} \tag{50}$$

There remains

$$\begin{aligned} J_\varepsilon(w_\varepsilon+\lambda w,z_\varepsilon)-J_\varepsilon(w_\varepsilon,z_\varepsilon) &= \frac{\lambda^2}{2}\left(\|w\|^2_{L^2(Q_\omega)}+\frac{1}{\varepsilon}\|w\chi_\omega\|^2_{L^2(Q)}\right) \\ &+\lambda\Big[\langle w_\varepsilon,w\rangle_{L^2(Q)}+\frac{1}{\varepsilon}\langle w_\varepsilon\chi_\omega,w\chi_\omega\rangle_{L^2(Q)} \\ &-\frac{1}{\varepsilon}\langle L^*z_\varepsilon-h_0\chi_O,w\chi_\omega\rangle_{L^2(Q)}\Big]. \end{aligned} \tag{51}$$

Passing to the limit, we get

$$\lim_{\lambda\longrightarrow 0}\left(\frac{J_\varepsilon(w_\varepsilon+\lambda w,z_\varepsilon)-J_\varepsilon(w_\varepsilon,z_\varepsilon)}{\lambda}\right)=0. \tag{52}$$

So

$$\langle w_\varepsilon,w\rangle_{L^2(Q)}-\frac{1}{\varepsilon}\langle L^*z_\varepsilon-h_0\chi_O-w_\varepsilon\chi_\omega,w\chi_\omega\rangle_{L^2(Q)}=0,\quad \forall w\in L^2(Q). \tag{53}$$

A calculation analogous to that above in equation (48) gives

$$\begin{cases}\dfrac{1}{\varepsilon}\displaystyle\int_Q L^*z(L^*z_\varepsilon-h_0\chi_O-\mathrm{w}_\varepsilon\chi_\omega)dxdt=0, & \forall z\in C^\infty(\bar{Q}) \text{ such that}\\ z=0 \text{ on } \Sigma, & z(T)=z(0)=0 \text{ in } \Omega.\end{cases} \tag{54}$$

We introduce the following adjoin state defined by

$$\rho_\varepsilon=-\frac{1}{\varepsilon}(L^*z_\varepsilon-h_0\chi_O-w_\varepsilon\chi_\omega). \tag{55}$$

So (53) and (54) become, respectively,

$$\begin{cases}\dfrac{1}{\varepsilon}\displaystyle\int_Q \rho_\varepsilon L^*z_\varepsilon dxdt=0, & \forall z\in C^\infty(\bar{Q}) \text{ such that}\\ z=0 \text{ on } \Sigma, & z(T)=z(0)=0 \text{ in } \Omega\end{cases} \tag{56}$$

and

$$\langle w_\varepsilon,w\rangle_{L^2(Q)}-\frac{1}{\varepsilon}\langle \rho_\varepsilon,w\chi_\omega\rangle_{L^2(Q)}=0,\quad \forall w\in L^2(Q_\omega). \tag{57}$$

We deduce

$$\langle \rho_\varepsilon,w\chi_\omega\rangle_{L^2(Q)}=-\langle w_\varepsilon,w\rangle_{L^2(Q)}. \tag{58}$$

From where

$$\mathrm{w}_\varepsilon=-\rho_\varepsilon\chi_\omega \text{ in } Q. \tag{59}$$

Thus, for z in $D(Q)$, we deduce that

$$L\rho_\varepsilon=0 \text{ in } Q. \tag{60}$$

On the other hand, $\rho_\varepsilon\in L^2(Q)$ with $L\rho_\varepsilon\in L^2(Q)$; then by application of the Lions-Magenes theorem, the trace function exists.

We multiply (60) by $z\in C^\infty(\bar{Q})$, and we integrate by part over Q; we get

$$\begin{aligned} 0&=\int_Q L\rho_\varepsilon z dxdt \\ &=\int_Q \rho_\varepsilon L^*z dxdt-\int_\Omega\rho_\varepsilon(0)z(0)dx+\int_\Omega\rho_\varepsilon(T)z(T)dx \\ &+\gamma\int_\Sigma\frac{\partial z}{\partial w}\rho_\varepsilon d\Sigma-\gamma\int_\Sigma\frac{\partial\rho_\varepsilon}{\partial w}z d\Sigma. \end{aligned} \tag{61}$$

In particular for $z\in C^\infty(\bar{Q})$ such that $z=0$ on Σ, $z(T)=z(0)=0$ in Ω and the fact that we have (54), then (61) becomes

$$\int_\Sigma\frac{\partial z}{\partial w}\rho_\varepsilon d\Sigma. \tag{62}$$

Finally,

$$\begin{aligned}\rho_\varepsilon&=0 \text{ in } \Sigma,\\ \rho_\varepsilon(0)&=0 \text{ in } \Omega.\end{aligned} \tag{63}$$

□

5.3. Optimality System (OS). In this subsection, we will state an important result which gives the optimality system for the problem (P), characterizing the optimal control, which also implies the construction of the sentinel.

Theorem 9. *The pair $(\widehat{w},\widehat{q})$ is the unique solution of (P), if and only if there exists a function $\widehat{\rho}$ such that the triplet $\{\widehat{w},\widehat{q},\widehat{\rho}\}$ is the solution of the following optimality system:*

$$\begin{aligned} &\widehat{w}\in L^2(Q_\omega),\ \widehat{q}\in H^{2,1}(Q),\ \widehat{\rho}\in V \\ &\begin{cases}-\partial_t\widehat{q}-\gamma\Delta\widehat{q}+\mu\widehat{q}=h_0\chi_O+\widehat{w}\chi_\omega, & \text{in } Q,\\ \widehat{q}(T)=0, & \text{in } \Omega,\\ \widehat{q}=0, & \text{on } \Sigma,\end{cases} \\ &\widehat{q}(0)=0, \text{ in } \Omega, \\ &\begin{cases}L\widehat{\rho}=0, & \text{in } Q,\\ \widehat{\rho}(0)=0, & \text{in } \Omega,\\ \widehat{\rho}=0, & \text{on } \Sigma,\end{cases} \\ &\widehat{w}=-\widehat{\rho}\chi_\omega. \end{aligned} \tag{64}$$

Proof. From the following equality,

$$\mathrm{w}_\varepsilon=-\rho_\varepsilon\chi_\omega, \tag{65}$$

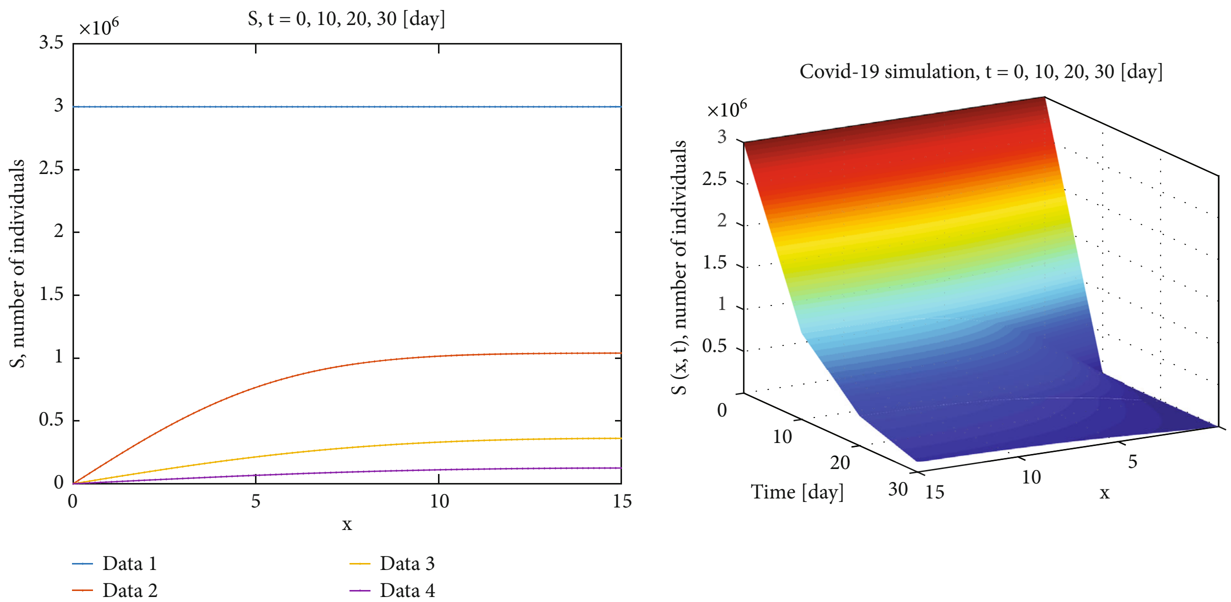

FIGURE 1: One section and 3-dimensional plot for the proportion of susceptible persons at risk of contracting COVID-19 in space and time.

and from the fact w_ε satisfies the inequality (33), it comes

$$\|\rho_\varepsilon \chi_\omega\|_{L^2(Q_\omega)} \le C, \tag{66}$$

and since

$$L\rho_\varepsilon = 0 \text{ in } Q, \tag{67}$$

we deduce that

$$\|\rho_\varepsilon\|_V^2 = \|L\rho_\varepsilon\|_{L^2(Q)}^2 + \|\rho_\varepsilon\|_{L^2(Q_\omega)} = \|\rho_\varepsilon \chi_\omega\|_{L^2(Q_\omega)}. \tag{68}$$

According to (66) and (68), we conclude that $(\rho_\varepsilon)_\varepsilon$ is bounded in V. Therefore, there are a subsequence $(\rho_\varepsilon)_\varepsilon$ and a function $\widehat{\rho}$ in V such that

$$\rho_\varepsilon \rightharpoonup \widehat{\rho} \text{ weakly in } V. \tag{69}$$

Thus,

$$\rho_\varepsilon \chi_\omega \rightharpoonup \widehat{\rho} \text{ weakly in } L^2(Q_\omega). \tag{70}$$

On the other hand, from the previous proposition, we have

$$w\varepsilon \rightharpoonup \widehat{w} \text{ weakly in } L^2(Q_\omega). \tag{71}$$

By uniqueness of the limit, we deduce that

$$\widehat{w} = -\widehat{\rho}\chi_\omega. \tag{72}$$

□

6. Information Given by the Sentinel

We assume that the population density N is observed on O, so

$$N_{\text{obs}} = m_0 \text{ on } O. \tag{73}$$

Because of (7), we can write

$$S(\lambda, \tau) - S(0,0) \approx \lambda \frac{\partial S}{\partial \lambda}(0,0), \quad \text{for } \lambda, \tau \text{ small}. \tag{74}$$

In (6), $S(\lambda;\tau)$ is observed and using (3)

$$S(\lambda;\tau) = \int_Q (h_0\chi_O + w\chi_\omega) m_0 \, dt\, da\, dx. \tag{75}$$

So (7) becomes

$$\lambda \frac{\partial S}{\partial \lambda}(0,0) \approx \int_Q (h_0\chi_O + w\chi_\omega)(m_0 - N_0)\, dt\, da\, dx, \tag{76}$$

where $y_0 = y(\lambda = 0, \tau = 0)$.

From (6), we have

$$\frac{\partial S}{\partial \lambda}(0,0) = \int_Q (h_0\chi_O + w\chi_\omega) N_\lambda \, dt\, da\, dx, \tag{77}$$

where here χ_O and χ_ω denote the characteristic functions of O and ω, respectively.

The derivative $N_\lambda = (\partial N/\partial\lambda(0,0))$ only depends on $\widehat{I}$ and other known data. Consequently, the estimate (76) contains the information on $\lambda\widehat{I}$ (see for details Remark 10 below).

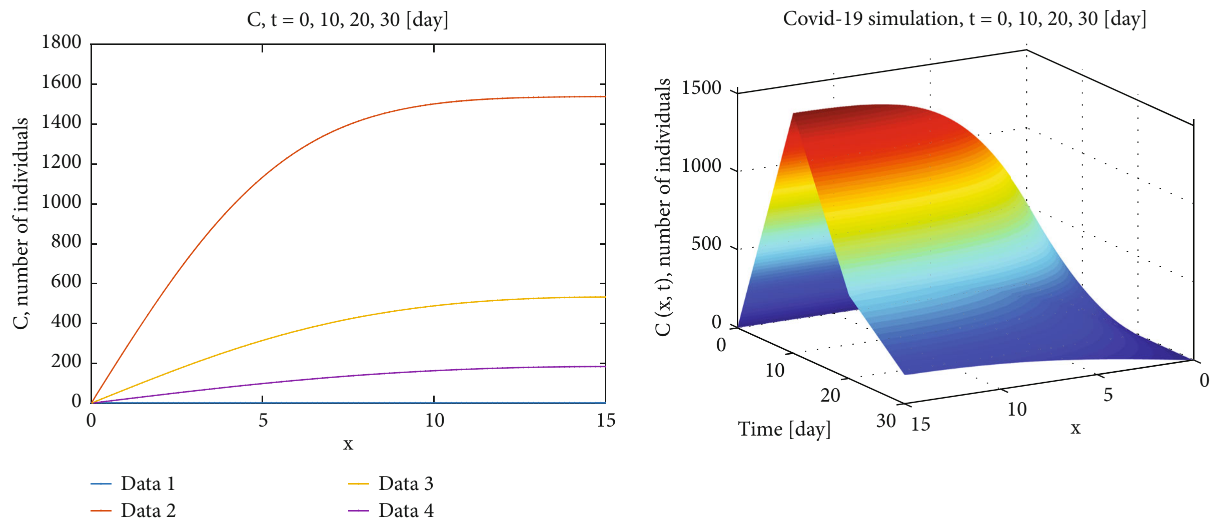

FIGURE 2: One section and 3-dimensional plot for the proportion of carriers (dead corpse) that transmit the COVID-19 in space and time.

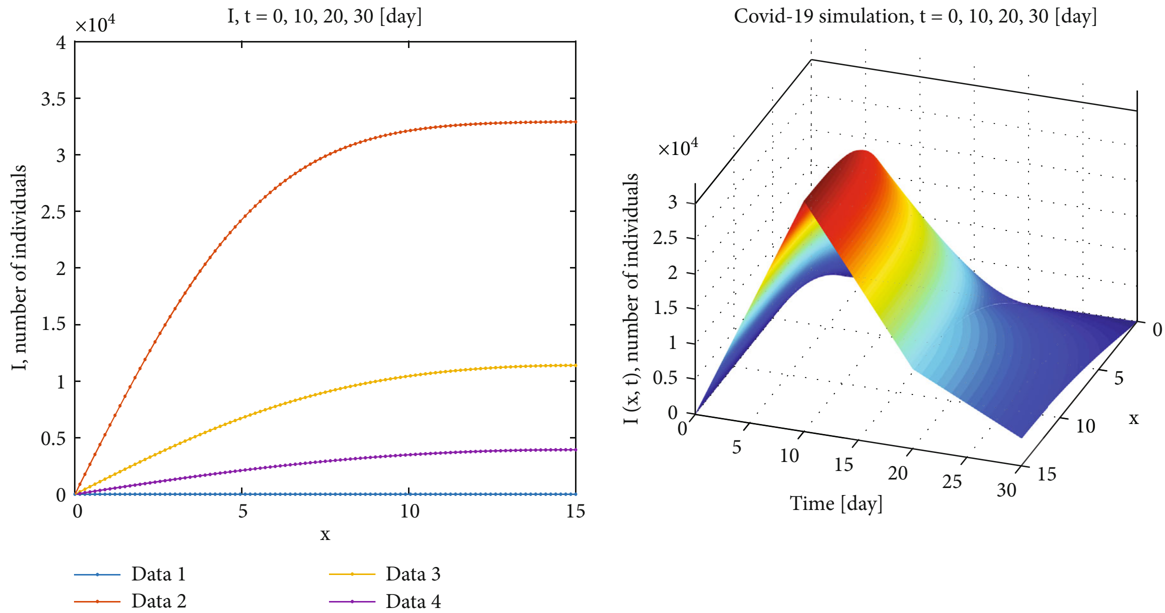

FIGURE 3: One section and 3-dimensional plot for the proportion infective persons capable of transmitting the COVID-19 to persons at risk in space and time.

Remark 10. The knowledge of the optimal control w provides informations about the pollution term $\lambda\widehat{I}$, and let $N_\lambda = \partial N/\partial\lambda(0,0)$ be the solution of

$$\begin{cases} \partial_t N_\lambda - \gamma \Delta N_\lambda + \mu N_\lambda = \widehat{I}, & \text{in } Q, \\ N_\lambda(0) = 0, & \text{in } \Omega, \\ N_\lambda = 0, & \text{on } \Sigma. \end{cases} \tag{78}$$

Multiplying (78) by q and integrating by parts over Q, we get

$$\int_Q q\widehat{I}dtdadx = \int_Q (h_0\chi_O + w\chi_\omega)N_\lambda dtdadx. \tag{79}$$

So from (16) and (77), we deduce

$$\int_Q \lambda\widehat{I}dtdadx = \int_Q (h_0\chi_O + w\chi_\omega)(m_0 - N_0)dtdadx. \tag{80}$$

7. Numerical Simulation of the Studies Model Solution

In this part, the idea is to highlight the numerical simulation of the solution of model (1), for $\lambda = \tau = 0$. The purpose is to present the method of lines (MOL) solution of the COVID-19 modeling equations, a system of five partial differential equations (PDEs), describing the interaction resulting between susceptible persons at risk of contacting COVID-19, carriers (dead corpse), infective persons, recovered

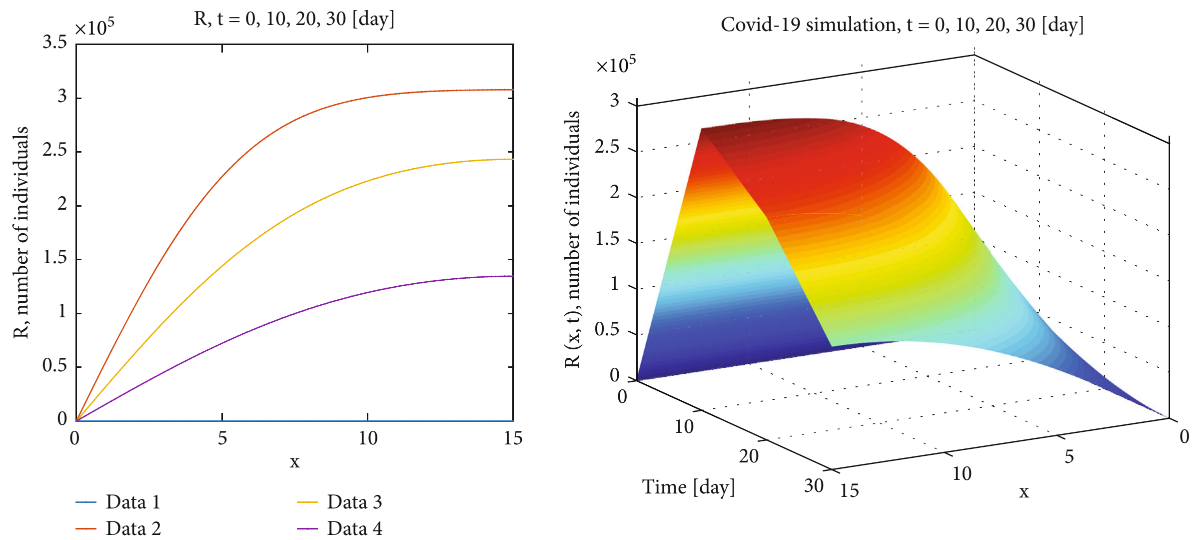

Figure 4: One section and 3-dimensional plot for the proportion of recovered persons who have been treated of COVID-19 in space and time.

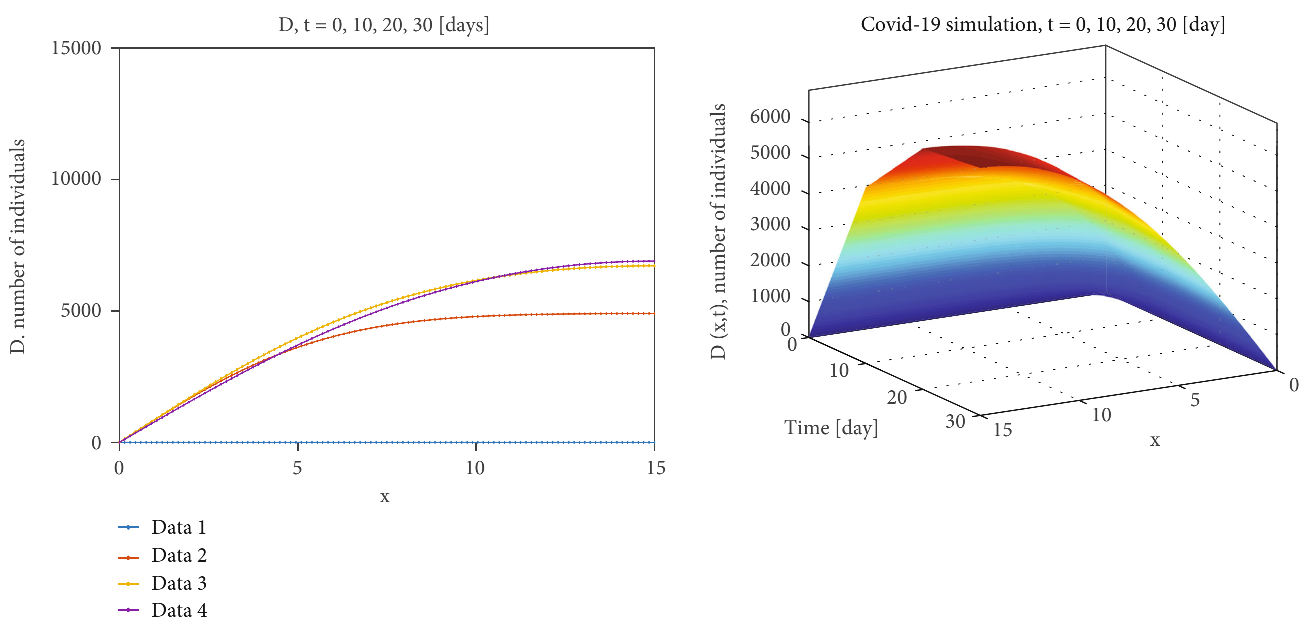

Figure 5: One section and 3-dimensional plot for the proportion of total number of deaths in space and time.

persons, and number of deaths. The method of lines is a semidiscrete method which involves reducing an initial boundary value problem to a system of ordinary differential equations (ODEs) in time through the use of a discretization in space. The resulting ODE system is solved by applying solver ode15s of MATLAB. This method provides very accurate numerical solution for linear or nonlinear PDE's in comparison with other existing methods. For initial conditions, let us look at the case of Ouagadougou city, Burkina Faso, where population is estimated to 3 million. Notice that Burkina Faso population is around 20 million. Towards the beginning of March 2020, the number of people infected was around 3. In order to make our model fit that data, the initial conditions (day zero) are $S(0) = 3000000$, $C(0) = 2$, $I(0) = 3$; $R(0) = 0$; $D(0) = 0$. One can note that parameter values are not for Burkina Faso excepted rate of natural death parameter. One must note that for all the following figures that the legend is data 1 for $t = 0$, data2 for $t = 10$, data 3 for $t = 20$, and data 4 for $t = 30$ Using the values of the parameters in Table 1, we obtain the following graphs.

Figure 1 shows that susceptible persons' number decreases progressively with a peak at $t = 10$ [day].

Figure 2 shows that the peak of the pandemic is reached on the tenth day, before beginning to decrease.

According to the numerical simulation of Figure 3, the number of infected persons evolves in the same manner as those of susceptible persons and carriers.

Figure 4 shows that number of people cured increases.

Figure 5 show that The number of deaths linked to COVID6-19 is decreasing.

The results are due to the fact that one does not have accurate data on COVID-19 death rate.

Remark 11. According to observations made after the numerical simulation of the COVID-19 model, it is very important to have real data that will allow us to do a good simulation with a good interpretation of the results in order to make them useful for decision makers.

8. Conclusion

In this paper, using a mathematical model of COVID-19 transmission, we have determined the number of people infected with COVID-19 by the sentinel method. Indeed, we first studied the existence and the construction of a sentinel for the system (3). We have shown that the search for a sentinel is indeed equivalent to the study of controllability. The null controllability is ensured by an observability inequality obtain through Carleman's estimates. Then, we used the constructed sentinel to obtain information on the number of people infected with COVID-19. Finally, we made a numerical simulation of the studied model solution. In perspective, in the concrete applications of singular nonlinear problems in epidemiology, the problem of controllability takes on its full meaning for epidemiological models. For example, consider the following system:

$$\begin{cases} \dfrac{\partial S}{\partial t} - \gamma \Delta S = \Lambda - \dfrac{\varepsilon S D}{N} - (\delta(x) + \mu) S + \eta R, & \text{in } Q, \\ \dfrac{\partial C}{\partial t} - \gamma \Delta C = \delta(x)\theta S + \dfrac{\varepsilon S D}{N} - (\beta + \mu + \pi) C, & \text{in } Q, \\ \dfrac{\partial I}{\partial t} - \gamma \Delta I = \delta(x)(1-\theta) S + \pi C - (\alpha + \mu + \sigma) I + v\chi_\omega, & \text{in } Q, \\ \dfrac{\partial R}{\partial t} - \gamma \Delta R = \beta C + \alpha I - (\mu + \eta) R, & \text{in } Q, \\ \dfrac{\partial D}{\partial t} - \gamma \Delta D = \sigma I, & \text{in } Q, \\ S(0) = S^0, & \text{in } \Omega, \\ C(0) = C^0, & \text{in } \Omega, \\ I(0) = I^0, & \text{in } \Omega, \\ R(0) = R^0, & \text{in } \Omega, \\ D(0) = D^0, & \text{in } \Omega, \\ S = 0, & \text{on } \Sigma, \\ C = 0, & \text{on } \Sigma, \\ I = 0, & \text{on } \Sigma, \\ R = 0, & \text{on } \Sigma, \\ D = 0, & \text{on } \Sigma, \end{cases} \tag{81}$$

as well as the functional

$$J(v) = \|I - z_d\|_{L^2(Q)} + N\|v\|_{L^2((0,T)\times\omega)}, \tag{82}$$

and we will then seek to determine the control v in such a way that it brings the number of infected to zero after a time T.

Data Availability

No underlying data was collected or produced in this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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